
STUDENTS' CRITICAL THINKING IN DETERMINING COEFFICIENTS OF ALGEBRAIC FORMS

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ABSTRACT

This paper reports the study of the critical thinking skills of 7th-grade students in junior high school on determining the coefficients of algebraic operation. Data were collected using worksheet and interviews. 34 students were involved during the study. The study reveals that there were 35% students who performed algebraic operations correctly. Students' difficulty is mainly on determining the coefficients associated with the algebraic forms. In general, errors occur in the process of finding algebraic solutions, explaining the reasons for the stages of strategies taken and drawing conclusions. This is due to the fact that the main attention was given to the final result of the process of working on algebraic form problems, and less focus was paid on giving reasons for the process of working on problems that stimulate the students to think critically.

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INTRODUCTION

Recently the problem with critical thinking has reappeared. It is been investigated intensively because critical thinking is significant in helping students solve their problems in real life (Sumarna et al., 2017). Mathematics is considered an important subject for improving students' ability in solving daily-life problems (Ngaeni & Saefudin, 2017). Furthermore, Rahmah (2013) argues that the systematic thinking process is the basic formation of mathematical concepts. Every student needs critical thinking skills competence for better mathematics learning (Agoestanto et al., 2016; Firdaus et al., 2015). Critical thinking includes several indicators, namely problem solving, formulating conclusions, calculating possibilities, and making decisions (Alifia et al., 2019). NCTM states that in developing student knowledge, teachers must be able to create an interesting learning environment not only to arrange information but also to review the relevance, usefulness, and interests of students in their lives, so students play an active role and can achieve mathematical skills especially in critical thinking (Kristianti et al., 2017). Realistic mathematics learning is considered an important method for improving the student's critical thinking skill (Palinussa, 2013).

In learning mathematics, a good understanding of algebra is unavoidably needed. Improving students' understanding and ability to use algebra is critical in the process of mathematics classrooms (Blanton et al., 2015). A similar argument was also proposed by Agoestanto et al., (2019) who state that the concept of algebra must be understood first before learning other mathematical concepts. This is supported by Apawu et al., (2018) who argue that in order to develop students' mathematical knowledge, their algebraic understanding must be developed first. Meanwhile, the essential part of learning algebra is to comprehend algebraic operations, namely addition, subtraction, multiplication and division. The sequence in the learning process of these operations should be given greater attention because the order cannot be exchanged (Jupri et al., 2019).

In general, algebra is an arithmetic generalization that uses symbols (variables) instead of number notation. Common difficulty faced by students in understanding the concept of algebra is due to the lack of understanding of the concept of variables (Lian & Idris, 2008). The use of various variable symbols causes students to have difficulty operating algebraic forms which causes difficulties in solving algebraic problems (Indraswari et al., 2018). Students' understanding of the concept of variables is vital to learn higher algebra, namely changing story

problems to mathematical models (algebraic forms). Paying close attention to the story problem and turning into a mathematical model requires students' critical thinking known as algebra critical thinking (Harti & Agoestanto, 2019). To improve the ability of higher level of thinking needs exercises to solve problems of daily life related to the concepts of algebra (Julius et al., 2018). Based on the above discussion, this paper focuses on investigating the 7th grade students' skills to use algebraic operations and an understanding of the coefficients in accordance with the algebraic form for developing their critical thinking.

METHOD

This non-experimental research was conducted by involving 35 students of 7th grade in a public school. The subjects were purposively selected due to their learning materials which were relevant to the present study, namely operation in algebraic form materials.

Two stages were carried out to obtain the data needed. As the first stage, the student's worksheet about the algebraic form and its operations were carefully examined to identify the students' ways of solving the given problems. To be more specific, the examination was done to check whether the students, under the study, use algebraic operations in accordance with the rules in mathematics or not. Based on the analysis of the student's worksheet, interviews were conducted with students in order to get more comprehensive data, especially the reasons why the students selected certain ways of algebraic operations. Interviews were conducted with the students who were classified into high, medium, and low groups based on their scores. The results were further discussed and recommendations were made for further studies.

RESULTS AND DISCUSSIONS

The results of the analysis are summarized in the following two tables. The first table summarizes the analysis on the student's responses to algebraic operations carried out by students on their worksheets. The second table summarizes the students' work on determining the coefficients on the variables that correspond to the algebraic form.

Table 1. The Students' Responses on Problem Operations in Algebra Forms ($9a^2b + 2a^2 + b^2 + 3ab^2 + 4ab + a^2b$)

Number	The Answer	N=34	%	Category	Average
1.	Obtained $10a^2b + 3ab^2 + 4ab + 2a^2 + b^2$	12	35%	True	35%
2.	Obtained $9a^2b + 7ab^2 + 2a^2 + b^2$	4	12%	False	
3.	Obtained $2a^2 + b^2 + 7ab^2 + 10a^2b$	9	26%	False	65%
4.	Obtained $9a^2b + 7ab^2 + 2a^2b$	7	21%	False	

Number	The Answer	N=34	%	Category	Average
5.	Obtained $10a^2b + 3ab^2 + 4ab + 2ab^2$	2	6%	False	

Analysis of student worksheets related to algebraic form operations is summarized in Table 1. Seen on student worksheet in simplifying forms $(9a^2b + 2a^2 + b^2 + 3ab^2 + 4ab + a^2b)$, here, there are as much as 65% of students who make errors. Interviews with students show the difficulties in carrying out algebraic operations. Students are not doing the stages of grouping the algebraic forms first, thus they are confused in seeing the bulky variables. This leads to student's confusion in seeing the appropriate coefficients of the algebraic form.

Table 2. The Students' Responses in Determining Coefficients for Variables a^2b and Variables b^2 in Problems $(9a^2b + 2a^2 + b^2 + 3ab^2 + 4ab + a^2b)$

Answer Category	N=34	%
Obtained $10a^2b + b^2$	21	62%
Obtained Only $10a^2b$	2	6%
Obtained Only b^2	4	12%
Average	27	79%

Table 2 shows that 21 students were able to determine the coefficients for the a^2b and b^2 variables in the problem $(9a^2b + 2a^2 + b^2 + 3ab^2 + 4ab + a^2b)$, it is explained that 21 students can determine the coefficients on the a^2b and b^2 variables with a percentage of 62%, 2 students were able to determine the coefficient but only on the a^2b variable with a percentage of 6%, and 4 students were able to determine the coefficient but only on the b^2 variable with a percentage of 12%.

The following examples represent the results of the observation reports of 3 students in answering questions given by the teacher with different answers.

Question : $9a^2b + 2a^2 + b^2 + 3ab^2 + 4ab + a^2b$

Student's answers :

$$\begin{aligned}
 \text{Student 1 : } & 9a^2b + 2a^2 + b^2 + 3ab^2 + 4ab + a^2b \\
 & = (9 + 0)a^2b + (3 + 4)ab^2 + 2a^2 + b^2 \\
 & = 9a^2 + 7ab^2 + 2a^2 + b^2
 \end{aligned}$$

$$\begin{aligned}
 \text{Student 2 : } & 9a^2b + 2a^2 + b^2 + 3ab^2 + 4ab + a^2b \\
 & = 2a^2 + b^2 + 3ab^2 + 4ab + a^2b + 9a^2b \\
 & = 2a^2 + b^2 + 7ab^2 + 10a^2b
 \end{aligned}$$

$$\begin{aligned}
 \text{Student 3 : } & 9a^2b + 2a^2 + b^2 + 3ab^2 + 4ab + a^2b \\
 & = 9a^2b + a^2b + 3ab^2 + 4ab + 2a^2 + b^2 \\
 & = 9a^2b + 7ab^2 + 2a^2b
 \end{aligned}$$

The unacceptable answers indicate that the students had difficulty in understanding the concept of algebra. As argued by Jupri & Drijvers (2016), the difficulty refers to obstacles such as implementing arithmetic operations, understanding the ideas of variables and algebraic expressions, and understanding different meanings of equal signs

From the interview script, it clearly showed how the students under the study made the flows of the processes they had gone through.

Researcher : How did you come to your answer $9a^2b + 2a^2 + b^2 + 3ab^2 + 4ab + a^2b$?

Student 1 : a^2b variable has coefficients 9 and 0 then it is added by $9 + 0$ to $9a^2b$ while variables a^2, b^2, ab^2 , and ab do not have the same term then it cannot be added.”

Researcher : How did you come to your answer $9a^2b + 2a^2 + b^2 + 3ab^2 + 4ab + a^2b$?

Student 2 : “because the ab variable has coefficients 3 and 4 then the sum of $3 + 4$ is $7ab^2$ and the a^2b variable has the coefficients 9 and 1 so the sums of $9 + 1$ to $10a^2b$ while the a^2 and b^2 variables do not have a similar term then it cannot be added up.”

Researcher : What is the term algebraic form of one banana + two bananas?”

Student 1 : “ $1p + 2p$ ”

Student 2 : “ $p + 2p$ ”

Researcher : How did you come to your answer?

Student 1 : “ p is banana so one banana is $1p$ and two bananas are $2p$ ”

Student 2 : “one banana is p and two bananas are $2p$ ”

Researcher : “Both answers are correct. In algebra, a variable or object or item whose number is 1, the coefficient is 1 but in terms of algebraic form no information is written 1 in front of the variable. Likewise in questions $9a^2b + 2a^2 + b^2 + 3ab^2 + 4ab + a^2b$, for the variable a^2b the coefficient is one and b^2 also

has a coefficient of 1 but no description is written 1 or symbol 1 in front of the variable, so it should be $9a^2b + 2a^2 + b^2 + 3ab^2 + 4ab + a^2b$

$$= (9 + 1)a^2b + 3ab^2 + 4ab + 2a^2 + b^2$$

$$= 10a^2b + 3ab^2 + 4ab + 2a^2 + b^2$$

and for $2a^2 + b^2 + 3ab^2 + 4ab$ it cannot be added up because apple and orange are different kinds of fruits.

CONCLUSION

From the analysis of the students' worksheets and the interviews, it can be concluded that the causes of students' difficulties are derived from (i) students' mastery of the concepts of algebraic forms is still lacking, (ii) students still do not fully understand the coefficient of algebraic forms, (iii) students are not yet sufficiently skillful to operate algebraic forms.

The students made algebraic errors due to their lack of understanding the notation of the variables used. They assumed that a^2b , ab^2 and ab are the same so that students are not able to do algebraic operations. To overcome this kind of problem, an illustration is necessary to be given through real objects so that they can understand how to differentiate these three forms. In fact, apple and banana can be denoted by a and b .

In addition, the students' understanding of coefficients needs more serious attention when learning algebraic forms. The findings revealed that it turned out that a coefficient of 1 that was not written explicitly made the students confused. The student did not understand that $1a$ was sufficiently written with a , $1ab$ was sufficiently written ab . Some students said that a^2b and b^2 had no coefficient and the coefficient values were zero even though, in general, the students well understood the coefficient values of more than 1, for example $2a$, $3ab$. The lack of understanding of these coefficients created errors when they performed algebraic operations. Once again, to overcome the students' difficulties, it is necessary to provide concrete examples as well as to understand the algebraic forms above.

The third problem is using algebraic operations. This problem emerged because students did not understand the form of algebra and the coefficient. As a result, they made mistakes in performing algebraic operations. For example, the students perceived the form $3a^2b + 2ab^2$ is equal to $5a^2b^2$. Furthermore, they also perceived that $3a^2b + 2ab^2 + a^2b + 5ab^2 = 3a^2b + 7ab^2$. This is because students believed a^2b has a coefficient of 0. The data indicates that the students

need more understanding the stages of grouping for the same forms of algebra. Thus, when working on the $3a^2b + 2ab^2 + a^2b + 5ab^2$ form it still requires the $(3a^2b + a^2b) + (2ab^2 + 5ab^2)$.

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