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# THE EFFECTIVENESS OF PROBLEM-BASED AND DISCOVERY LEARNING MODELS ON STUDENTS' MATHEMATICAL CRITICAL THINKING ABILITIES

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***ABSTRACT***

Critical thinking skills are essential for addressing 21st-century challenges, which requires a learning model that can optimally engage students' cognitive abilities. This study aims to examine the improvement of students' critical mathematical thinking skills (KBKM) through learning that utilizes the Problem-Based Learning (PBL) and Discovery Learning models, based on the students' skill levels. The method used in this study is a mixed-method approach with a sequential explanatory design. In the quantitative phase, quasi-experimental research was conducted with a one-group pretest-posttest design, ANOVA, and a 3x2 factorial design. In the qualitative phase, an embedded concurrent strategy was applied. The sample consisted of 72 grade VII students from State Junior High Schools in Cimahi City. The results revealed that the criteria had a significant effect on the critical mathematical thinking skills scores. The learning model also had a direct impact on improving these skills. Additionally, the study found that the average score of the high group was higher than that of the medium and low groups. In terms of learning models, Discovery Learning proved to be more effective in the learning process.

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## INTRODUCTION

In the 21st century, critical thinking skills have become an essential core competency that enables learners to address complex global challenges and solve real-world problems in an increasingly data-driven world (Arsih et al., 2023). Learning mathematics is crucial in fostering analytical, problem-solving, and logical reasoning skills, which are essential for individuals to thrive in contemporary society (Angraini & Nurmaliza, 2022). However, traditional mathematics teaching methods, which are often teacher-centered and repetitive, have been criticized for not adequately developing the advanced cognitive abilities needed in modern learning environments (Suryawan et al., 2023).

To address these shortcomings, constructivist approaches such as problem-based learning (PBL) and discovery learning (DL) have become widely utilized to engage students and strengthen critical thinking abilities in mathematics (Palinussa et al., 2023). While PBL focuses on real-world problem-solving opportunities that encourage students to actively explore and apply knowledge, DL emphasizes the active discovery of concepts through exploration and reflective analysis (Akhir et al., 2023). Despite the growing interest in these strategies, there have been no direct empirical investigations into their relative benefits for promoting mathematical critical thinking (Suparman et al., 2021).

These abilities include various aspects needed to address challenges in the current era, such as critical thinking, problem-solving, communication, and creativity (Kids, 2019). In the context of mathematics learning, developing mathematical proficiency is essential. Kilpatrick et al., (2002) identify four main components of mathematical skills that need to be developed: conceptual understanding, procedural fluency, adaptive reasoning, and productive disposition. In Indonesia, students' critical mathematical thinking skills are still not optimally developed. identified that the level of this skill is relatively low across various levels of education. This finding is supported by several studies conducted at different school levels (Angraini & Nurmaliza, 2022; Benyamin et al., 2021; Demir, 2022; Dewi & Maulida, 2023; Septian & Komala, 2019; Shahrill et al., 2018; Warsah et al., 2021). Problem-based learning (PBL) encourages student engagement in solving authentic problems that require comprehensive analysis and collaborative efforts, thereby directly training critical and reflective thinking skills (Ghifari et al., 2024; Hung, 2009; Rahayu et al., 2023; Sulistyawati et al., 2023). Discovery Learning (DL) provides students with opportunities for independent exploration, analysis, and discovery of new concepts, thus strengthening logical and creative thinking abilities (Alfieri et al., 2011). By combining these methodologies within the learning framework, it is hoped that students will achieve an optimal level of development in their critical thinking skills in mathematics.

Discovery Learning shares many similarities with Problem-Based Learning (PBL). Both approaches focus on problem-based learning and require students to analyze, explore, and find solutions independently and collaboratively. Therefore, Discovery Learning and Problem-Based Learning can be considered complementary in supporting the development of students' mathematical critical thinking skills.

This research presents a novel approach by juxtaposing two learning models that emphasize independent learning. Based on the background and the urgency of the issue, the aim of this research is to compare the effectiveness of problem-based learning and discovery learning models in improving students' mathematical critical thinking skills.

## METHOD

Since this research aims to examine the improvement of students' mathematical critical thinking abilities (KBKM) through learning using the Problem-Based Learning (PBL) and Discovery Learning models, based on the students' levels of mathematical critical thinking ability, a quasi-experimental method was used as the quantitative methodology in this study (Dharma et al., 2020; Suteja & Setiawan, 2022). The research employed a mixed-methods approach with an explanatory sequential design. In the quantitative phase, quasi-experimental research was applied using a one-group pretest-posttest design, one-way ANOVA, and a 3x2 factorial design. In the qualitative phase, an embedded concurrent strategy design was utilized. The sample in this study consisted of 72 students from grade VII at State Middle Schools in Cimahi City. The quasi-experimental design was chosen because the classes in the school were naturally occurring, and therefore, the sample was not re-randomized. This design is consistent with recommendations from previous studies evaluating similar educational interventions (Dharma et al., 2020; Sucilestari & Arizona, 2020).

This study focused on the seventh-grade junior high school student population across all public junior high schools in Cimahi City, as critical thinking skills in mathematics begin to develop significantly at the junior high school level (Rahman et al., 2022). The sampling technique used was non-probability sampling with a purposive sampling method. The research applied the Problem-Based Learning (PBL) model and the Discovery Learning model. The social cognitive learning model syntax applied in this study includes the following stages: (1) attention, (2) retention, (3) production, and (4) motivation (Bandura, 1977). The problem-based learning model syntax consists of the following stages: (1) problem presentation, (2) problem investigation, (3) solution generation, and (4) evaluation (Awang & Ramly, 2008; Soden, 1994)

The development of mathematical critical thinking ability test items includes the following aspects (see Table 1): Ennis (Elliott et al., 2001) identified the following indicators: 1) Focus, 2) Reason, 3) Inference, 4) Situation, 5) Clarity, and 6) Overview.

Table 1. Mathematical Critical Thinking Ability Indicators

| Aspect | Indicator |
|---|---|
| Focus | Focusing on decision-making based on existing problems |
| Reason | Providing rational reasons for decisions made |
| Inference | Drawing conclusions based on evidence while considering alternatives |
| Situation | Understanding the key problems that lead to certain situations |
| Clarity | Explaining the meaning of terms in the problem |
| Overview | Rechecking the accuracy of decisions made |

## RESULT AND DISCUSSION

Table 2. Two-way ANOVA test

| Source | Type III Sum of Squares | df | Mean Square | F | Sig. |
|---|---|---|---|---|---|
| Corrected Model | 1.493[a] | 5 | .299 | 84.889 | .000 |
| Intercept | 20.599 | 1 | 20.599 | 5856.849 | .000 |
| Criteria | 1.312 | 2 | .656 | 186.570 | .000 |
| Learning Model | .035 | 1 | .035 | 9.849 | .003 |
| Criteria * Learning Model | .003 | 2 | .001 | .396 | .675 |
| Error | .232 | 66 | .004 | | |
| Total | 31.114 | 72 | | | |
| Corrected Total | 1.725 | 71 | | | |

a. R Squared = .865 (Adjusted R Squared = .855)

Based on Table 2, the two-way ANOVA test reveals that the criteria have a significant effect on the scores, with an F value of 186.570 and a significance of 0.000 ($p<0.05$). This indicates that the average scores in each category (high, medium, and low) are different. Additionally, the learning model factor showed a significant effect with a p-value of 0.003 ($p<0.05$), suggesting that the use of different learning models resulted in differences in student scores. However, the interaction between the criteria and learning model on the scores was not significant ($F = 0.396$; $p = 0.675$), so it can be concluded that the effect of criteria on scores is independent of the learning model, and vice versa. This analysis model has an $R^2$ of 0.865, meaning that 86.5% of the variation in scores can be explained by the combination of criteria factors, learning models, and their interactions. This value indicates that the model used in this study is highly effective in explaining the variation in scores.

Table 3. Post hoc Tukey test

| (I) Criteria | (J) Criteria | Mean Difference (I-J) | Std. Error | Sig. | 95% Confidence Interval Lower Bound | 95% Confidence Interval Upper Bound |
|---|---|---|---|---|---|---|
| High | Medium | .1917* | .01553 | .000 | .1544 | .2289 |
| | Low | .4125* | .02083 | .000 | .3625 | .4624 |
| Medium | High | -.1917* | .01553 | .000 | -.2289 | -.1544 |
| | Low | .2208* | .01984 | .000 | .1732 | .2684 |
| Low | High | -.4125* | .02083 | .000 | -.4624 | -.3625 |
| | Medium | -.2208* | .01984 | .000 | -.2684 | -.1732 |

Based on observed means.
The error term is Mean Square(Error) = .004.
*. The mean difference is significant at the 0.05 level.

Based on Table 3, the results of the further test using the Tukey test on the criterion variables revealed significant differences between the three criterion groups (high, medium, and low) in terms of their scores. The analysis results show that the average score of the high group was significantly higher than both the medium and low groups, with differences of 0.1917 and 0.4125, respectively. Meanwhile, the average score of the medium group was also significantly higher than the low group, with a difference of 0.2208. All of these differences were found to be significant ($p = 0.000 < 0.005$), indicating that the higher the students' abilities, the higher the scores they obtained.

Figure 1. Problem Based Learning Learning Activities.

Figure 1 shows that the activities of students learning with the PBL model have the following characteristics: student-centered learning, collaborative learning, and active involvement among students (Ghifari et al., 2024; Rahayu et al., 2023; Safitri et al., 2022; Sulistyawati et al., 2023). Student-centered learning means that students play a significant role in the learning process, including formulating problems, collecting information, analyzing data, and finding solutions. Collaborative learning involves students working in small groups to solve problems, sharing knowledge, and developing communication and cooperation skills. Active student involvement requires engagement in the learning process, starting with formulating problems, gathering information, analyzing data, and producing solutions.

Studies conducted by (Hidayati & Listyani, 2010; Lestari, 2013) add that the PBL approach can improve critical thinking skills by fostering deeper analytical abilities in understanding mathematical concepts. This is further supported by findings from (Sung et al., 2016), which show that the use of PBL helps students develop reflective thinking skills essential for solving complex problems.

Figure 2. Learning Activities with Discovery Learning.

Figure 2 shows an active learning atmosphere with the Discovery Learning approach, where students work in groups to explore and understand the material independently. They are seen reading books, writing, and discussing, reflecting the steps in Discovery Learning: stimulation, problem identification, data collection, proof, and generalization. This method encourages students to discover concepts independently through exploration and collaboration, thereby improving critical thinking skills, problem-solving abilities, and collaboration in learning.

Based on the two-way ANOVA results shown in the test of between-subjects effects (see Table 2), it can be concluded that both the criteria factors and the learning model have a significant effect on the scores, while the interaction between the two factors is not significant. Several factors influence these results. Regarding the influence of criteria on scores, the criteria factor shows a very significant impact on scores ($F = 186.570$, $p = 0.000$). This means that the average score differs among the High, Medium, and Low criteria categories. In line with (Slavin, 2012), who states that differences in students' abilities or initial criteria are an important factor in achieving learning outcomes, students with higher levels of critical thinking skills tend to perform better academically than students with lower critical thinking skills.

Regarding the influence of learning models on scores, the learning model factor has a significant impact on scores (F = 9.849, p = 0.003), meaning that the learning model influences students' achievement of learning outcomes. This aligns with (Joyce & Weil, n.d.), who emphasized that choosing the right learning model can optimize students' conceptual understanding and development of thinking skills. The differences between Discovery Learning and Problem-Based Learning models provide different results. Based on the average score of each learning model, the results of statistical tests show that the Discovery Learning model is more effective than the Problem-Based Learning model. According to (Bruner, 1961), Discovery Learning helps students be active and construct their own knowledge, making the learning process more meaningful and improving students' abilities. According to (Bloom, 1976) success in learning depends on the prerequisites students have before participating in the learning process. The higher the students' initial abilities, the more optimal their results.

## CONCLUSION

This study concludes that the criteria for mathematical critical thinking skills (high, medium, and low) and the learning model each directly influence mathematical critical thinking skills. However, there is no interaction effect between the criteria and the learning model. Additionally, the group of students with high criteria demonstrates more development in achieving mathematical critical thinking skills. In terms of the learning model, Discovery Learning is found to be a more effective approach in the learning process.

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