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# Exploring cognitive pathways of mathematics education students in solving contextual modular arithmetic problems

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## ABSTRACT
Understanding how university students grasp modular arithmetic in real-world contexts remains underexplored, particularly regarding their use of cognitive pathways when solving contextualized problems. This study aims to examine the cognitive pathways utilized by mathematics education students in solving context-based modular arithmetic problems. A total of 32 undergraduate students in the early stages of a mathematics education program, who had successfully completed a course in number theory, were asked to solve three real-world problems involving work cycle patterns, container multiples, and congruence systems. The tasks were designed to represent fundamental real-world structures of modular arithmetic and to explore how contextual representations influence students' cognitive pathways. The analysis was conducted using a descriptive qualitative approach through Cognitive Task Analysis (CTA) to identify five key cognitive pathways: quantitative reasoning, linguistic processing, working memory, pattern recognition, and cognitive flexibility. The results show that the first three pathways were consistently used across all tasks, indicating stable patterns in numerical reasoning, language processing, and information management. In contrast, pattern recognition and cognitive flexibility varied depending on how the problems were presented. Contextual narratives encouraged more reflective and adaptive thinking, whereas symbolic forms tended to limit exploration and reduce conceptual engagement. These findings highlight the importance of problem design in supporting diverse cognitive approaches. The study's practical implications include the development of assessments and learning strategies that foster flexible and meaningful thinking, particularly in understanding abstract mathematical concepts. Future research is encouraged to explore the integration of digital technologies and multiple representations to enhance students' cognitive flexibility.

## KEYWORDS
Cognitive pathways, contextual problems, modular arithmetic

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## INTRODUCTION

Modular arithmetic is a fundamental branch of number theory with strong relevance to various real-world applications, including calendar systems, modern cryptography, information coding, scheduling, and control systems. Its core concept—number congruence—refers to the relationship between two numbers that yield the same remainder when divided by a given modulus (Rosen, 2011). Despite its essential and applicable nature, modular arithmetic is often taught procedurally in higher education, with an emphasis on symbolic manipulation rather than fostering deep conceptual understanding (Schüler-Meyer, 2019; Barumbun & Kharisma, 2022). Rittle-Johnson (2017) asserts that a balance between procedural skills and conceptual understanding is essential for supporting meaningful, long-term learning. Similarly, Borji et al. (2019) found that instruction grounded in conceptual understanding enhances both problem-solving retention and flexibility more effectively than purely algorithmic approaches.

Instruction that focuses solely on step-by-step procedures may therefore be insufficient for cultivating a comprehensive understanding of mathematical structures, particularly in applied contexts. Students exposed primarily to algorithmic methods often fail to develop transferable thinking skills or adaptive reasoning (Rittle-Johnson, 2017; Borji et al., 2019). An overemphasis on procedural fluency can limit opportunities for strategic exploration and weaken connections between abstract ideas and real-world experience (Shen et al., 2024; Schüler-Meyer, 2019).

One promising pedagogical approach involves integrating real-world contextual problems that connect abstract mathematical concepts to tangible situations. Context-based learning enables students to engage with real-life scenarios meaningfully, promoting reflective and adaptive strategies as well as flexible thinking. For instance, Amalia et al. (2024) demonstrated that contextual problem design enhances cognitive engagement and improves mathematical problem-solving skills. Similarly, Kohen and Nitzan-Tamar (2022) and Chavarría-Arroyo and Albanese (2022) highlight that such tasks foster thinking flexibility, deepen learning outcomes, and support adaptive cognitive strategies.

Previous research on mathematics learning has predominantly focused on problem-solving outcomes rather than the cognitive processes underlying students' reasoning (Verschaffel et al., 2020; Muhtasyam et al., 2024; Shen et al., 2024). Evidence from mathematical cognition suggests that performance arises from the interaction of cognitive resources such as working memory, representational processing, and strategic decision-making—factors that outcome-based assessments alone cannot fully capture (Raghubar et al., 2010; Verschaffel et al., 2020). Recent findings further emphasize that mathematical reasoning involves encoding semantic and real-world representations during problem solving, underscoring the importance of examining internal cognitive processes beyond observable performance (Gros et al., 2024). This points to a critical gap in understanding how students process contextual mathematical information, particularly in complex domains such as modular arithmetic. Investigating the cognitive pathways activated during problem solving is therefore essential for informing pedagogical interventions that promote conceptual understanding and flexible thinking.

LeFevre et al. (2010) conceptualized cognitive pathways as key routes underlying individual differences in mathematical development, encompassing quantitative processing, linguistic competence, working memory, pattern recognition, and cognitive flexibility.

Subsequent research has refined this framework, showing that the influence of these pathways varies across developmental stages and learning contexts rather than serving as fixed predictors of performance (Sowinski et al., 2015; Träff et al., 2019). More recent studies highlight the integration of domain-general and domain-specific processes in complex mathematical reasoning, emphasizing flexible representational coordination in adaptive problem solving (Scheibling-Sève et al., 2017; Jaffe & Bolger, 2023). Collectively, these developments position cognitive pathways as interacting processes that support adaptive mathematical reasoning across contexts.

The framework proposed by LeFevre et al. has been widely applied to examine the development of numerical literacy in children and adolescents, particularly through pathway-based analyses of arithmetic performance, strategy use, and representational processing in diverse instructional settings (LeFevre et al., 2010; Sowinski et al., 2015; Träff et al., 2019). Nevertheless, its application in studies involving mathematics education students remains limited, especially in contexts requiring complex and reflective engagement with contextual problems. While some studies have explored pre-service teachers' numerical thinking (e.g., Subekti et al., 2022; Getenet, 2022), they have not explicitly integrated LeFevre's cognitive pathway framework. This gap underscores the need for further investigation into how mathematics education students formulate cognitive strategies and navigate challenges within each pathway.

This study aims to conduct an in-depth examination of the cognitive pathways that mathematics education students use when solving modular arithmetic problems presented in real-life contexts. The primary focus is on the thinking patterns that emerge during problem solving, as well as the cognitive strategies students employ to address the tasks. By systematically analyzing these processes, this study seeks to provide a more comprehensive understanding of how students construct meaning around modular arithmetic concepts. The findings are expected to inform the development of more adaptive, student-centered mathematics instruction—particularly for abstract topics—by emphasizing thinking processes over mere procedural proficiency.

## METHODS

This study employed a descriptive research design with a qualitative approach (Creswell & Creswell, 2023), using Cognitive Task Analysis (CTA) to examine the cognitive pathways utilized by mathematics education students when solving contextual modular arithmetic problems. The study focuses on the thinking processes that emerge during problem solving rather than solely on the correctness of final answers. CTA was selected because it is well suited to investigating complex mathematical tasks situated in real-world contexts and allows for an in-depth exploration of underlying cognitive dynamics. This approach facilitates the identification of expert thinking patterns and the implicit knowledge activated in specific situations (Tofel-Grehl & Feldon, 2013; Brown, Power, & Gore, 2022), thereby supporting the development of more accurate and reflective instructional design.

The study involved 32 undergraduate students in the early stages of a mathematics education program who had successfully completed a course in number theory. Participants were selected through purposive sampling to ensure that they possessed sufficient foundational knowledge to engage meaningfully with the tasks. Successful completion of the Number Theory course provided evidence of their understanding of congruence, factorization, and modular number systems, as indicated by prior academic performance. This foundational knowledge was essential for examining the cognitive pathways employed in solving contextual modular arithmetic problems, as the study aimed to analyze reasoning processes rather than basic concept acquisition.

The researchers developed three contextual modular arithmetic problems, each intentionally designed to activate specific cognitive pathways. The design was guided by five primary pathways identified in prior research—quantitative processing, linguistic processing, working memory, pattern recognition, and cognitive flexibility—which formed the analytical framework of the study, with their respective indicators presented in Table 1. This framework was synthesized from multiple sources. LeFevre et al. (2010) initially proposed the quantitative, linguistic, and working memory pathways, emphasizing numerical fluency, number sense, and the capacity to store and manipulate complex information. Sowinski et al. (2015) and Jaffe and Bolger (2023) highlighted the importance of linguistic strategies in interpreting and translating narrative-based problems into mathematical representations. Träff et al. (2019) demonstrated the contributions of working memory and pattern recognition, underscoring the importance of identifying numerical cycles and general structures as predictors of mathematical success. Scheibling-Sève et al. (2017) emphasized the role of cognitive flexibility, reflecting the ability to shift strategies or adjust approaches when initial attempts prove unsuccessful.

These five pathways were operationally adapted to the context of university students solving modular arithmetic problems, ensuring that the tasks elicited reasoning processes across multiple cognitive pathways.

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**Table 1.** Cognitive Pathway Indicators

| Code | Cognitive Pathway | Indicators |
|---|---|---|
| QT | Quantitative | Proficiency in arithmetic and understanding of numerical relationships |
| LG | Linguistic | Ability to read, interpret, and understand context-based narrative problems |
| WM | Working Memory | Capacity to store and manage complex information within an integrated solution |
| PT | Patterning | Recognition of numerical patterns, cycles, or general structures |
| CF | Cognitive Flexibility | Ability to switch strategies and adjust approaches when initial attempts fail |

Following their development, the problems underwent expert validation. Two experts in mathematics and mathematics education evaluated each item for its relevance to the research objectives, appropriateness of difficulty, and potential to engage the targeted cognitive pathways. This process ensured that all problems were suitable for eliciting the intended cognitive processes. The three contextual modular arithmetic problems and their corresponding codes are presented in Table 2.

**Table 2.** Contextual Modular Arithmetic Problems

| Code | Problem |
|---|---|
| Q1 | A nurse works on a rotating schedule of 4 working days followed by 2 days off. The cycle repeats consistently. She begins her first workday on Monday, May 5, 2025. On which upcoming date will she next return to work on a Monday? |
| Q2 | A factory wants to deliver goods using three types of containers:<br>● Container A can hold items in multiples of 10 units.<br>● Container B in multiples of 15 units.<br>● Container C in multiples of 25 units.<br>What is the minimum number of items that must be shipped so that all three container types can be fully loaded without any remaining items? Explain the process using prime factorization and the concept of congruence. |
| Q3 | A factory assigns serial numbers to its products based on the remainders of divisions by three production machines:<br>● Machine X: remainder 2 when divided by 3<br>● Machine Y: remainder 1 when divided by 5<br>● Machine Z: remainder 4 when divided by 7<br>The product must be given the smallest possible positive serial number that satisfies all three conditions. Determine the smallest serial number that satisfies the following system:<br>$x \equiv 2 \pmod 3$, $x \equiv 1 \pmod 5$, $x \equiv 4 \pmod 7$ |

Data analysis was conducted using a qualitative Cognitive Task Analysis (CTA) approach, emphasizing the identification and mapping of the cognitive processes students employed during problem solving (Crandall & Hoffman, 2013; Tofel-Grehl & Feldon, 2013; Brown, Power, & Gore, 2022). Student responses were collected through written solutions to the three modular arithmetic problems.

All responses were systematically coded according to predefined cognitive pathway indicators (see Table 1). The coding captured students' strategies, reasoning steps, and alternative approaches. The coded data were then organized into a student–pathway table. This structure enabled the visualization of both individual and collective reasoning patterns, as well as the interaction between cognitive pathways and problem characteristics.

The table was analyzed to identify recurring strategies, examine variations in pathway activation, and explore alternative approaches. Particular attention was given to how narrative-based versus symbolic problem formats influenced the use of different pathways. To quantify pathway engagement, a scoring rubric (see Table 3) was applied, operationalizing observable behaviors for each cognitive pathway while maintaining a focus on reasoning processes rather than the correctness of final answers.

**Table 3.** Scoring of Cognitive Pathway

| Cognitive Pathway | Scoring Guidelines |
|---|---|
| Quantitative | Students perform calculations such as prime factorization, modulo operations, LCM, or other solution-finding strategies involving numbers. |
| Linguistic | Students understand word-based (non-symbolic) problems and translate them into mathematical expressions. |
| Working Memory | Students construct logical, step-by-step solutions for complex or multi-step problems. |
| Patterning | Students identify repeated patterns, cycles, or multiples and use these patterns to simplify or predict processes. |
| Cognitive Flexibility | Students attempt alternative strategies and switch approaches when the initial one fails or proves inefficient. |

## RESULT AND DISCUSSION

This section presents the findings on the cognitive pathways demonstrated by mathematics education students when solving contextual modular arithmetic problems. Written responses were coded according to five cognitive pathways, and each problem was analyzed to determine levels of engagement and variations in individual cognitive patterns.

### *Result*

#### *Cognitive Pathways in Problem One (Q1)*

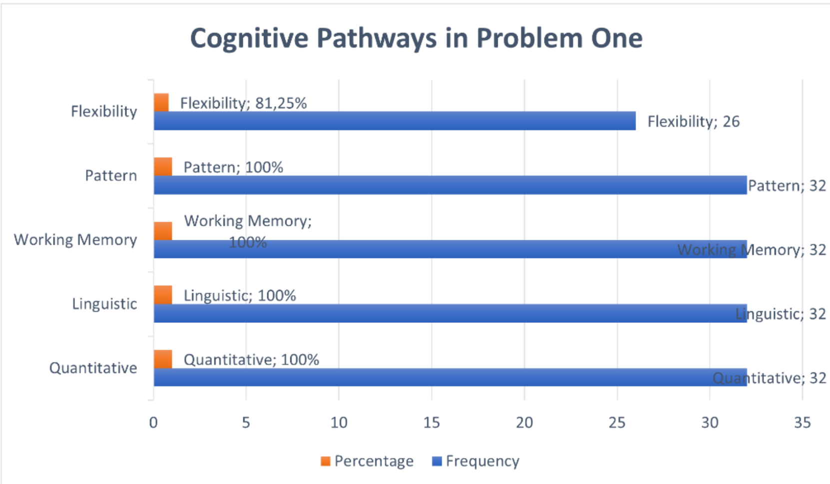


**Figure 1.** Cognitive Pathways in Q1

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Figure 1 presents the distribution of cognitive pathways demonstrated by students when solving the first contextual modular arithmetic problem.

The results indicate that most students activated a wide range of cognitive pathways while solving the problem. Of the 32 participants, 26 students (81%) demonstrated all five cognitive pathways: QT, LG, WM, PT, and CF. These students successfully interpreted the contextual narrative, determined the length of the work cycle, tracked the sequence of workdays and rest days, identified the recurring structure of the schedule, and organized their reasoning in a systematic manner.

Six students (19%) demonstrated four pathways. In these responses, cognitive flexibility was not evident. Although these students correctly interpreted the contextual information, performed the necessary calculations, and recognized the repeating cycle, their solutions relied on a single procedural strategy without attempting alternative representations or approaches.

Examples of these reasoning patterns are presented in Figure 2. The first sample illustrates a solution that engages all five cognitive pathways. The student organized the sequence of workdays and rest days, identified the repeating cycle, and traced the calendar progression until the schedule aligned with Monday. The second sample represents a response demonstrating four pathways. In this case, the student correctly interpreted the schedule and performed the calculations but relied on a single procedural strategy without attempting alternative approaches or verification.

The comparison of responses in Figure 2 reveals how pathway engagement shapes students' reasoning. Students who activated all five pathways constructed more integrated and flexible solutions, effectively combining contextual interpretation with numerical analysis. In contrast, students demonstrating four pathways tended to follow a fixed, linear approach, focusing on sequential calculations and showing limited adaptation or strategic variation.

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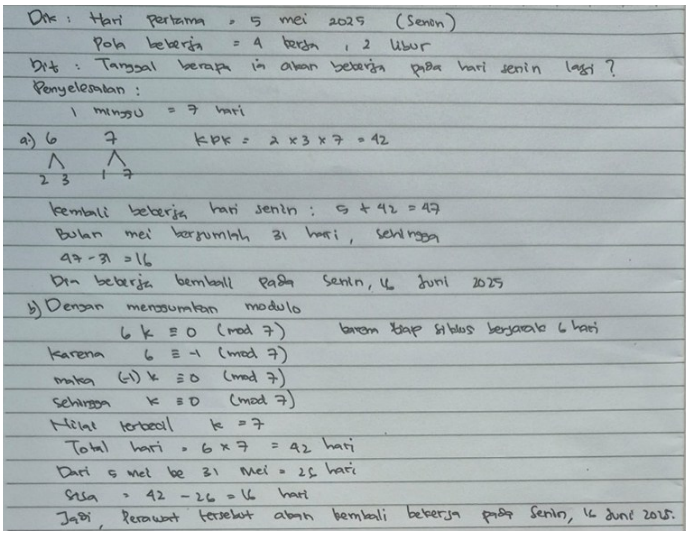
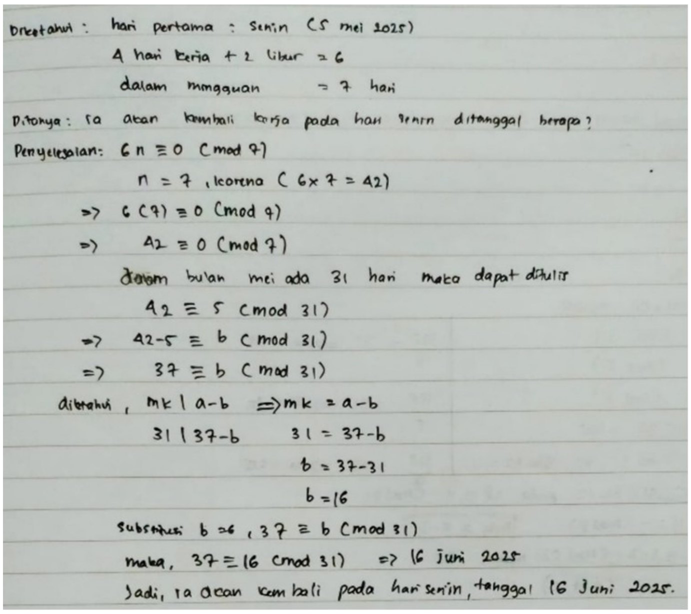



**Given:** First day = May 5, 2025 (Monday)
Work pattern = 4 days working, 2 days off
**Question:** On what date will he work again on a Monday?

Return to working on Monday: 5 + 42 = 47
May has 31 days, so:
47 − 31 = 16
He will work again on Monday, June 16, 2025.

Smallest value: k = 7
Total days = 6 × 7 = 42 days
From May 5 to May 31 = 26 days
Remaining: 42 − 26 = 16 days
So, the employee will work again on Monday, June 16, 2025.




**Given:** First day: Monday (May 5, 2025)
4 days working + 2 days off = 6 days
In one week = 7 days
**Question:** On what date will he work again on a Monday?

In the month of May there are 31 days, so it can be written:

Therefore, he will return to work on Monday, June 16, 2025.

**Figure 2.** Sample Answers from Students with Different Cognitive Approaches in Q1

## Cognitive *Pathways* in Problem Two (Q2)

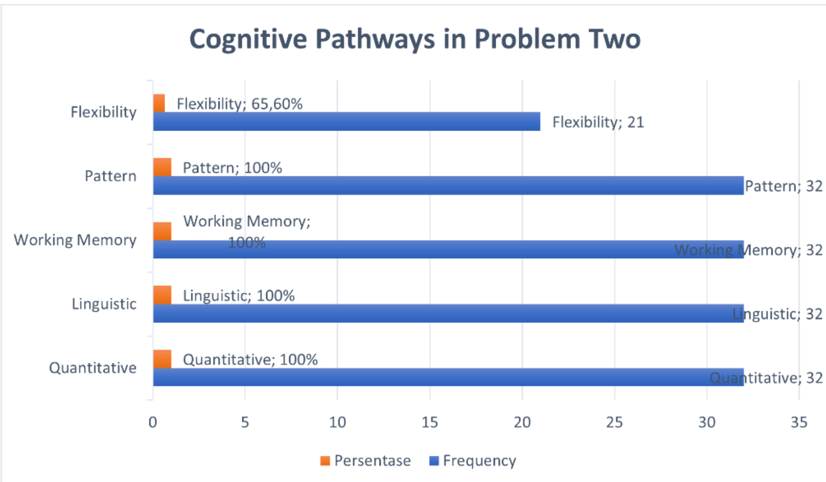



**Figure 3.** Cognitive Pathways in Q2

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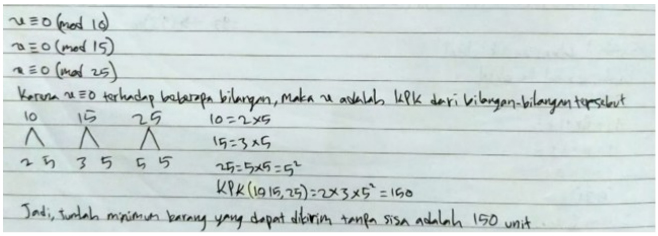
Figure 3 illustrates the distribution of cognitive pathways used in solving Q2, which required students to determine the smallest number of items that could be evenly distributed among three types of containers. Compared with the first task, engagement of cognitive pathways was slightly lower. A total of 21 students (65.6%) demonstrated all five pathways. These students correctly interpreted the contextual conditions, applied numerical procedures such as prime factorization or least common multiple (LCM) calculations, and organized the solution process in a structured sequence. The remaining 11 students demonstrated four pathways, with cognitive flexibility again being the most frequently absent pathway. In these responses, students typically relied on a single strategy, such as listing multiples of each container capacity until a common value was identified. Although these students achieved the correct result, their solutions showed no indication of strategy modification.

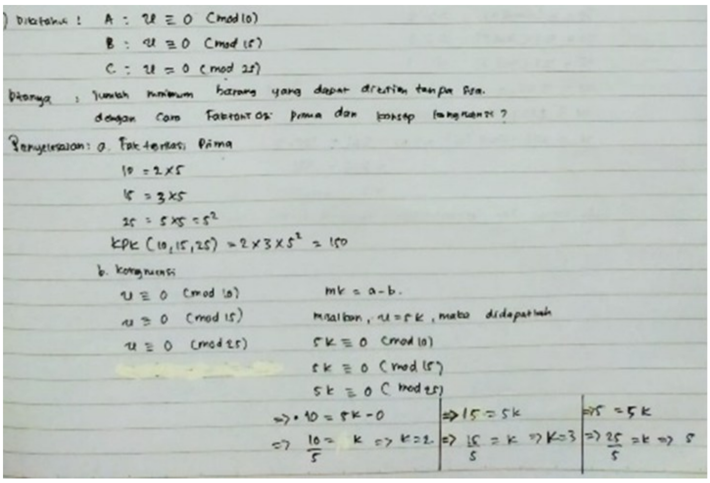



Because n≡0 for all these numbers, then n is the LCM (Least Common Multiple) of those numbers.

Therefore, the minimum number of goods that can be shipped without remainder is 150 units.

**Figure 4.** Student Responses Demonstrating Four Pathways in Q2

Representative responses are shown in Figure 4 and Figure 5. Figure 4 demonstrates a four-pathway response, in which the student obtained the correct answer solely by determining the LCM, without using modular reasoning as a verification step. Figure 5 illustrates the activation of all five cognitive pathways. In this solution, the student calculated the least common multiple (LCM) and then verified the result using modular reasoning, reflecting careful evaluation of the solution from an additional mathematical perspective.




Question: What is the minimum number of goods that can be distributed without any remainder?

**Figure 5.** Student Responses Demonstrating Five Pathways in Q2

The examples in Figure 4 dan Figure 5 reveal how the presence of cognitive flexibility

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affects students' strategic reasoning. Students who activated all five pathways combined the use of the least common multiple with modular verification to confirm the correctness of their solutions. This combination indicates more adaptable reasoning. In contrast, students who demonstrated only four pathways relied on a single procedural strategy, focusing narrowly on sequential calculations and showing limited adaptation or extension of their reasoning.

*Cognitive Pathways in Problem Three (Q3)*

The results for Q3 are shown in Figure 6. This task required students to determine the smallest positive integer satisfying a system of modular congruence conditions.

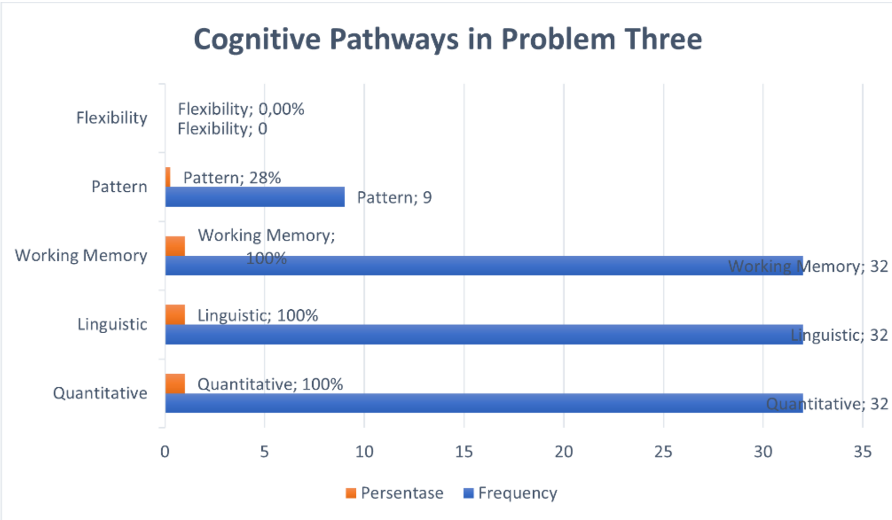


**Figure 6.** Cognitive Pathways in Problem Three

In comparison with the previous problems, engagement of cognitive pathways was more limited. None of the students demonstrated the simultaneous use of all five pathways. A total of 23 students demonstrated four pathways, while nine students demonstrated only three pathways.

Among the four-pathway responses, pattern recognition was not observed. Students translated the verbal conditions into modular expressions and applied substitution procedures to test values that satisfied the given congruence relationships. However, their written solutions did not indicate attempts to identify recurring numerical structures. The nine students who demonstrated three pathways exhibited a more restricted reasoning process. In these responses, both pattern recognition and cognitive flexibility were absent. Students typically followed a single sequential procedure, testing possible values until the correct solution was found.

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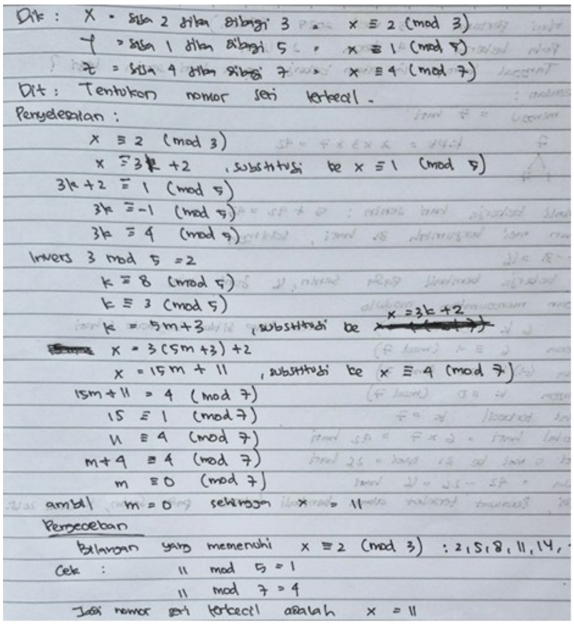
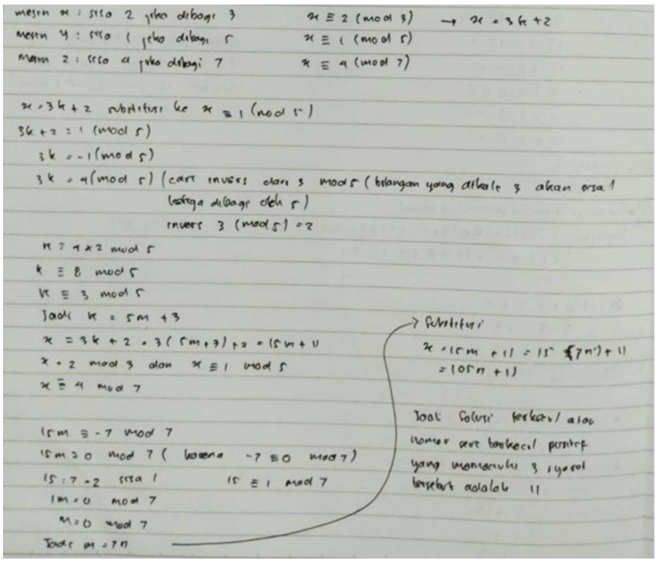


Given:
$x$ leaves a remainder of 2 when divided by 3 → x≡2 (mod 3)
$y$ leaves a remainder of 1 when divided by 5 → x≡1 (mod 5)
$z$ leaves a remainder of 4 when divided by 7 → x≡4 (mod 7)
Question: Find the smallest seat number.

Check (trial):
Numbers satisfying $x$≡2 (mod 3): 2, 5, 8, 11, 14, …
Check:11 mod 5 = 111 mod 7 = 4
So, the smallest seat number is $x$=11.



Machine $x$ leaves 2 if divided by 3 x≡2 (mod 3) → x=3k+2
Machine $y$ leaves 1 if divided by 5 x≡1 (mod 5)
Machine $z$ leaves 4 if divided by 7 x≡4 (mod 7)

(find the inverse of 3 mod 5 (a number which, when multiplied by 3 , leaves a remainder of 1 when divided by 5))

Therefore, the smallest solution, or the smallest positive integer that satisfies all three conditions, is 11.

**Figure 7.** Sample Answers from Students with Different Cognitive Approaches in Q3

Examples of these responses are presented in Figure 7. The first sample illustrates a four-pathway response, in which the student solved the problem without demonstrating pattern recognition. The second sample represents a three-pathway response, lacking both pattern recognition and cognitive flexibility and relying solely on sequential substitution to reach the solution.

The comparison of responses in Figure 7 highlights the impact of reduced cognitive pathway engagement on students' reasoning. When fewer pathways were activated, solutions became more procedural and narrowly focused. The absence of pattern recognition and cognitive flexibility limited students' ability to identify structural relationships within the modular conditions, resulting in reliance on step-by-step substitution rather than exploration of

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underlying numerical patterns or alternative strategies.

### *Discussion*

The findings of this study demonstrate that students' engagement with cognitive pathways varied according to the structure and representation of the modular arithmetic problems. Across all three tasks, quantitative reasoning, linguistic processing, and working memory were the most consistently activated pathways, whereas pattern recognition and cognitive flexibility showed greater variation depending on the problem context.

Quantitative reasoning appeared in all student responses across Q1–Q3. Students consistently applied numerical procedures such as calculating work cycles, determining least common multiples, and testing values in modular systems. This pattern aligns with the Pathways to Mathematics framework proposed by LeFevre et al. (2010), which identifies quantitative processing as a central component of mathematical reasoning. Sowinski et al. (2015) further emphasize that quantitative reasoning plays a key role in recognizing numerical structures and relationships in mathematical tasks, while Träff et al. (2019) similarly demonstrate that arithmetic performance is strongly shaped by cognitive mechanisms related to numerical processing.

The linguistic pathway also played a significant role, particularly in Q1 and Q2, where students had to interpret contextual narratives. Understanding terms such as "cycle," "days off," and "even distribution" was essential for translating textual information into mathematical operations, as observed in the sample solutions presented in Figures 2 and 4. These findings align with Jaffe and Bolger (2023), who emphasize that successful mathematical problem solving requires integrating linguistic processing with numerical reasoning. Research on word problems further indicates that the clarity and structure of language influence students' ability to form accurate mathematical representations (Vessonen et al., 2024). Students who effectively combined linguistic interpretation with quantitative calculations tended to produce more integrated and systematic solutions, as demonstrated by responses activating all five pathways in Q1 and Q2.

Working memory was consistently engaged across all tasks, as students needed to manage multiple pieces of information simultaneously, including workday sequences, intermediate factorization results, and modular conditions. This demand was most evident in Q3, where symbolic congruence required tracking multiple remainders and test values. These findings are consistent with Raghubar et al. (2010) and Medrano and Miller-Cotto (2025), who

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show that working memory supports the maintenance of intermediate results and coordinated execution of multi-step arithmetic procedures. In this study, effective use of working memory was observable in responses that organized steps coherently, particularly among students activating four or five pathways.

The engagement of pattern recognition varied with problem representation. In Q1, the rotating work schedule encouraged the identification of a repeating six-day cycle, and the results in Figure 2 confirm that most students used this pathway effectively. By contrast, pattern recognition was less frequently observed in Q3, which was presented primarily in symbolic form (Figure 6). These results support findings by Träff et al. (2019) and Stylianou (2011), suggesting that visual or contextual cues facilitate pattern recognition more effectively than purely symbolic representations. Responses lacking pattern recognition tended to rely on sequential substitution, as shown in the three-pathway examples for Q3, indicating more procedural and narrowly focused reasoning.

Cognitive flexibility was the least consistently observed pathway. While several students demonstrated flexibility in Q1 by reorganizing information or attempting multiple solution approaches, this ability declined in Q2 and was largely absent in Q3. Many students relied on a single procedural strategy even when alternative approaches were feasible, as reflected in the four- and three-pathway responses shown in Figures 4 and 6. This observation aligns with Rittle-Johnson (2017) and Scheibling-Sève et al. (2017), who note that learners often default to familiar procedures unless instructional contexts encourage exploration of alternative strategies.

The differences across the three tasks indicate that problem representation influences students' cognitive engagement. Contextual problems promoted broader involvement, including interpretation, pattern recognition, and strategic adaptation, whereas primarily symbolic tasks tended to encourage procedural reasoning. These findings are consistent with Schüler-Meyer (2019), who found that students transitioning to tertiary mathematics often rely on procedural manipulation in modular arithmetic tasks.

Overall, the findings indicate that students' engagement with cognitive pathways varied depending on the problem type. These findings align with research indicating that contextual mathematical abilities develop differently from symbolic skills, requiring coordination of linguistic interpretation, quantitative reasoning, and working memory (Shen et al., 2024; Gilmore et al., 2015). The observed differences in pathway activation highlight the influence of task structure and representation on students' reasoning strategies.

Overall, the findings demonstrate that solving contextual modular arithmetic problems engages multiple cognitive pathways. Quantitative reasoning, linguistic processing, and working memory consistently supported problem solving, whereas pattern recognition and cognitive flexibility were more sensitive to the contextual complexity of each task. These results highlight the importance of designing learning environments and assessment tasks that incorporate varied problem representations, fostering engagement across diverse reasoning strategies and promoting a more comprehensive understanding of mathematical concepts. This study contributes to a clearer understanding of how students coordinate multiple cognitive pathways when solving contextual modular arithmetic problems, showing that effective problem solving relies on their simultaneous engagement rather than solely on numerical accuracy or procedural execution.

## CONCLUSION

This study explored five cognitive pathways—quantitative reasoning, linguistic processing, working memory, pattern recognition, and strategic flexibility—used by mathematics education students in solving context-based modular arithmetic problems. The first three pathways were consistently applied across tasks, reflecting stable use of numerical strategies, language comprehension, and information management. In contrast, pattern recognition and strategic flexibility depended on problem representation. Context-rich narrative tasks fostered more reflective and adaptive thinking, whereas symbol-heavy problems limited strategic exploration.

These findings highlight the critical role of problem design in shaping the diversity of students' thinking approaches. The results suggest that assessment and instruction should be developed to accommodate a broader range of cognitive pathways beyond procedural approaches. Future research is encouraged to investigate how digital technologies and multiple representations enhance flexibility in information processing and strengthen students' conceptual understanding of abstract mathematical ideas in more meaningful ways.

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