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Geo-Sheet: Interactive GeoGebra-integrated learning material to enhance students’ creative mathematical reasoning in analytic geometry

Titin Masfingatin, Ika Krisdiana, and Reza Kusuma Setyansah

Mathematics Education Study Program, Universitas PGRI Madiun, Madiun, Indonesia

ABSTRACT

In analytic geometry instruction, students’ reasoning often remains procedural, limiting opportunities to develop Creative Mathematical Reasoning (CMR). CMR refers to the ability to construct reasoning that is novel to the learner, justified as plausible, and grounded in intrinsic mathematical properties. This study aimed to design and evaluate *Geo-Sheet*, a GeoGebra-integrated interactive learning material intended to support students’ CMR in analytic geometry. The research involved 30 undergraduate students enrolled in an Analytic Geometry course in a Mathematics Education program at a private university in East Java, Indonesia. An Educational Design Research (EDR) approach was employed, consisting of preliminary and formative evaluation phases (self-evaluation, expert review, one-to-one, small group, and field testing). Geo-Sheet was assessed in terms of validity, practicality, and potential effect. Expert judgment indicated high validity (Aiken’s V = 0.82), and student questionnaires showed high perceived practicality (70% in the high category). Pre- and post-test results, supported by observations and interviews, revealed improvements across CMR indicators. These findings suggest that systematically designed GeoGebra-based materials that integrate dynamic visualization and guided exploration have the potential to create productive learning conditions for the development of higher-order mathematical reasoning.

KEYWORDS

GeoGebra; creative mathematical reasoning; analytic geometry; mathematics education

CORRESPONDING AUTHOR

Titin Masfingatin ✉ titin.mathedu@unipma.ac.id 🖃 Mathematics Education Study Program, Universitas PGRI Madiun, Madiun, Indonesia

INTRODUCTION

Mathematical reasoning is a fundamental competency in mathematics education. For prospective teachers, reasoning skills support their ability to interpret, develop, and respond to students' mathematical thinking (Haji & Yumiati, 2019; Hamidah et al., 2025). Reasoning plays a central role in mathematics because concepts are constructed through logical arguments, not merely from observation (Agusti et al., 2023). It reflects students' ability to connect concepts and justify their conclusions (Hidayat, 2017; Marsitin et al., 2022).

One important form of reasoning for meaningful learning is Creative Mathematical Reasoning (CMR), defined as the ability to construct original solutions rather than simply imitate existing procedures (Lithner, 2008, 2017; Masfingatin & Murtafiah, 2020; Rahmah et al., 2024). CMR is characterized by students’ ability to construct learner-novel reasoning sequences using alternative representations or strategies, justify the plausibility of these choices by coherently linking algebraic procedures with geometric meanings, and ground their reasoning in the intrinsic mathematical properties of geometric objects and their analytic representations. It requires students to formulate strategies, connect concepts, and critically evaluate solutions (Lithner, 2017; Olsson & Granberg, 2024).

However, in practice, many students still rely on imitative reasoning, reproducing worked examples without understanding their conceptual structure (Agusti et al., 2023; Hidayat et al., 2018; Rahmah et al., 2024), This leads to shallow understanding and difficulty in solving non-routine problems. This pattern is consistent with findings from the Programme for International Student Assessment (PISA 2022), which indicate that Indonesian students' reasoning performance remains concentrated at Levels 1–2 and is largely associated with routine procedural tasks (OECD, 2023). These findings suggest that weaknesses in mathematical reasoning are not limited to school contexts but may persist into higher education.

Research on preservice teachers indicates that their creative mathematical reasoning varies in terms of their ability to construct and justify original solution strategies (Palengka et al., 2019). These challenges are particularly evident in analytic geometry, where studies report that preservice mathematics teachers experience difficulties in developing creative mathematical thinking, especially in tasks requiring flexible connections between algebraic and geometric representations (Hidayat et al., 2024; Yildiz et al., 2017). This situation highlights the need for instructional designs that intentionally promote flexible, exploratory, and justification-oriented reasoning processes rather than procedural imitation.

One promising approach to addressing these issues is the development of technology-integrated teaching materials that deliberately stimulate students' reasoning. Interactive materials provide opportunities for students to explore, test ideas, and actively construct knowledge, thereby supporting reasoning and problem-solving processes (Atmaja et al., 2021; Listiani et al., 2024; Olsson & Granberg, 2024). Previous studies have emphasized the importance of visual and dynamic representations in fostering deep conceptual understanding (Masfingatin & Murtafiah, 2020) and recommend integrating technology to encourage creative reasoning.

GeoGebra, as a dynamic mathematics software, offers simultaneous algebraic and geometric representations, immediate feedback, and opportunities for exploration, which enhance students' understanding of geometric concepts and support their mathematical thinking (Yildiz et al., 2017; Zutaah et al., 2023). These affordances enable students to explore relationships between symbolic and visual forms, which are essential for developing deeper mathematical insight. However, a systematic review by Yohannes & Chen (2021) found that most GeoGebra-based studies primarily focus on improving conceptual understanding and performance, with less emphasis on fostering reasoning through intentionally designed instructional materials. Similarly, Sunzuma (2023) reported that technology integration in geometry learning has largely emphasized effectiveness and learning outcomes, while relatively few studies explicitly address the development of mathematical reasoning. This trend reflects a broader emphasis in recent literature on the use of digital tools and visual representations to support learning (Jablonski & Ludwig, 2023).

Despite the recognized potential of GeoGebra to support mathematical thinking and exploration in analytic geometry (Khalil et al., 2019), limited attention has been given to how GeoGebra-integrated instructional materials can be systematically designed to foster Creative Mathematical Reasoning, in which students actively construct, justify, and refine their own solution strategies based on mathematical relationships.

Therefore, this study introduces *Geo-Sheet*, an interactive GeoGebra-integrated instructional material specifically designed to foster students' Creative Mathematical Reasoning in analytic geometry. Geo-Sheet was developed based on guided inquiry theory to encourage active student engagement in knowledge construction (Casey & Hallissy, 2014; Moreno & Mayer, 2007), as well as the principles of CMR to support the independent construction of solutions (Granberg & Olsson, 2015; Jonsson et al., 2022; Lithner, 2017; Olsson et al., 2024). These principles facilitate generative processing, which underpins creative reasoning. The novelty of this study lies in the design of interactive teaching materials that integrate dynamic visualization with guided inquiry principles.

This paper describes the design and development of GeoGebra-based teaching materials (*Geo-Sheet*) using an Educational Design Research approach. The study aims to develop and evaluate Geo-Sheet in terms of validity, practicality, and potential effect in supporting CMR in analytic geometry. In line with Nieveen (1999), quality is defined through three criteria: validity, practicality, and potential effect. A teaching material is considered valid when it is grounded in current scientific knowledge and relevant to its context and objectives (Plomp & Nieveen, 2013). It is practical when it can be implemented effectively in the intended learning context. Potential effect refers to its capacity to support students' achievement of intended learning outcomes, particularly in fostering CMR. A detailed description of the design research process is provided, along with an analysis of how the developed materials support students' reasoning.

METHODS

This study employed Educational Design Research (EDR) following the framework proposed by Plomp and Nieveen (2013), which aims to systematically develop and refine educational interventions through iterative cycles to achieve validity, practicality, and potential effectiveness. Formative evaluation is particularly recommended in the development of digital teaching materials to ensure systematic and progressive refinement prior to broader implementation (Mertasari & Candiasa, 2022; Pertiwi et al., 2023).

The research was conducted at a private university in East Java and involved undergraduate students enrolled in an Analytic Geometry course within the Mathematics Education program. In the preliminary phase, a needs analysis was conducted through classroom observations and analysis of students' work involving 42 students to identify learning difficulties and reasoning characteristics. Participants were involved sequentially across the formative evaluation stages.

In the one-to-one tryout phase, two students were selected to assess the readability and comprehensibility of the material. This was followed by a small-group evaluation involving six students to examine usability and instructional flow. Finally, the field test was conducted in one intact class comprising 30 undergraduate students (N = 30), all of whom had previously participated in the preliminary needs analysis, to examine the practicality and potential effect of the intervention.

The research consisted of three main phases: preliminary, prototyping, and assessment. The progression across phases reflects increasing resistance to change, where early stages allow more flexible revisions, while later stages require more selective refinement due to contextual complexity.

Preliminary Phase

During the preliminary phase, a needs analysis was conducted to identify learning problems, student characteristics, supporting facilities, and material requirements in the Analytic Geometry course (Hidayat et al., 2022; Purwitaningrum & Prahmana, 2021). Data were collected through classroom observations, analysis of students' written work and achievement records, and semi-structured interviews.

Observations were carried out during regular lectures using a structured observation sheet to examine instructional practices, students' participation patterns, and the types of reasoning employed in problem solving. Students' written responses and achievement data were analyzed to identify performance levels and reasoning tendencies. Semi-structured interviews with selected students explored their problem-solving approaches, difficulties in constructing conceptual justifications, and their familiarity with and use of GeoGebra in learning activities.

The data were analyzed descriptively to characterize students' needs, material conditions, and available learning infrastructure. These findings served as the basis for designing the initial prototype (Prototype I) of the Geo-Sheet digital teaching materials.

Prototyping Phase

The prototyping phase implemented iterative formative evaluation within the broader EDR framework proposed by Plomp and Nieveen (2013). The sequence of formative evaluation was adapted from Tessmer's (1993) model, comprising self-evaluation, expert review, one-to-one tryout, and small-group testing. This structure enables systematic refinement of the prototype prior to broader classroom implementation.

Self-evaluation was conducted by the researchers to examine internal consistency, content organization, and technical functionality of Prototype I. The prototype was then reviewed by four experts: two material experts in geometry instruction and two media experts experienced in digital mathematics learning design. The experts evaluated the teaching materials using a validation sheet covering content suitability, presentation (construction), language, and alignment with principles of teaching material development. Feedback from the experts was used to revise Prototype I.

Subsequently, a one-to-one tryout was conducted with two students representing the target population to identify clarity issues, usability concerns, and potential misconceptions at the individual level. Revisions were made based on findings from both the expert review and the one-to-one tryout, resulting in Prototype II.

The revised prototype (Prototype II) was then implemented in a small group consisting of six students with heterogeneous abilities (high, medium, and low). This stage aimed to examine the practicality and usability of the teaching materials in a limited classroom setting. Revisions at this stage focused on refinement based on user experience and interaction patterns, resulting in Prototype III, which was prepared for field testing.

Consistent with the concept of resistance to change, the stages within the prototyping phase represent relatively low-resistance conditions, as revisions could still be implemented flexibly before exposure to full classroom complexity. Through these iterative cycles, progressively improved prototypes were produced prior to the assessment phase.

Assessment Phase

The assessment phase consisted of a field test involving 30 students enrolled in the Analytic Geometry course. The field test aimed to evaluate the practicality and potential effect of the developed Geo-Sheet teaching materials in an authentic classroom setting. Due to real classroom dynamics, revisions after the field test were selective and focused on refinement rather than structural redesign. The findings served as the basis for establishing the final version of Geo-Sheet.

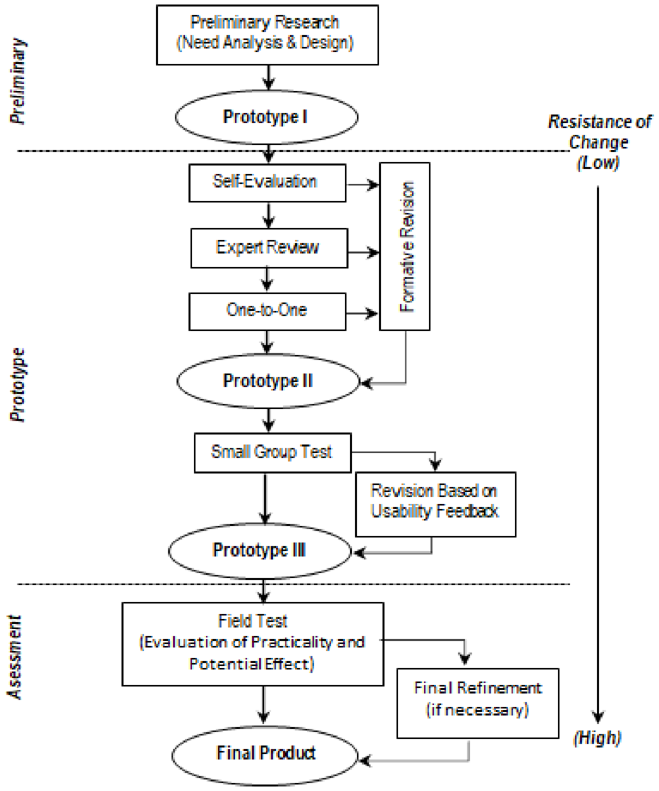


Figure 1. Formative Evaluation Design adapted from Aulia and Prahmana (2022)

Figure 1 presents the overall formative evaluation framework guiding the development process, illustrating iterative prototype refinement across the preliminary, prototyping, and assessment phases, along with increasing resistance to change toward the final product.

Data Analysis

This section outlines the data analysis procedures based on the Educational Design Research criteria of validity, practicality, and potential effect. Both quantitative and qualitative data were analyzed to evaluate the quality of the Geo-Sheet prototype and its effectiveness in supporting students' Creative Mathematical Reasoning.

Validity

Validity data were obtained from expert evaluations using a validation sheet consisting of 12 indicators covering content suitability, presentation, language, and alignment with teaching material development principles. The instrument employed a five-point Likert scale. Aiken's V index was calculated for each indicator, and the average index was used to determine the overall validity level (Retnawati, 2016).

Practicality

Practicality data were collected through a student response questionnaire administered during the small-group and field test stages. The questionnaire consisted of 17 statements measured using a five-point Likert scale and covered three aspects: perceived effectiveness, efficiency, and creativity. Practicality levels were interpreted according to the categorization criteria proposed by Azwar (2010).

Potential Effect

The potential effect of Geo-Sheet in stimulating students' Creative Mathematical Reasoning (CMR) was examined during the field test. Data were collected through pre-test and post-test results, classroom observations, and student interviews. Quantitative data were analyzed descriptively to identify changes in students' performance across three CMR indicators: novelty, plausibility, and mathematical foundation. Qualitative data were analyzed to provide deeper insights into the development of students' reasoning processes during Geo-Sheet activities.

RESULT AND DISCUSSION

This section presents the results of the study and discusses them in relation to the research objectives. The findings are organized according to the stages of Educational Design Research, beginning with the preliminary results of the needs analysis, followed by the outcomes of the design and formative evaluation phases, and concluding with the potential effect identified during the field test.

Preliminary Result

The development of the Geo-Sheet interactive teaching materials began with a needs analysis. At this stage, student needs were analyzed based on identified learning problems, student characteristics, material content, and available learning support facilities. The needs analysis involved 42 students enrolled in the Analytic Geometry course, as presented in Table 1.

The results of the needs analysis presented in Table 1 informed the subsequent design stage. Based on these findings, a Geo-Sheet prototype was designed by integrating a digital worksheet with a GeoGebra applet on the topic of circles. The design process began with structuring student-centered content to promote independent exploration, followed by the development of a responsive interface optimized for access via laboratory computers and students' smartphones. This stage resulted in Prototype I.

At this stage, validation instruments were constructed to assess the feasibility of the prototype. Supporting research instruments, including observation sheets, a student response questionnaire, and test items to measure students' CMR abilities, were also developed.

Table 1. The Results of Needs Analysis

No	Aspect	Results of observations and interviews
1.	Learning process	- Classroom observations during Analytic Geometry lectures showed that instruction on the circle topic relied primarily on PowerPoint presentations containing static GeoGebra screenshots rather than interactive exploration. - Observations of classroom interactions indicated that learning was predominantly teacher-centered, with limited opportunities for students to discuss, justify, or explore alternative solution strategies. - Analysis of students' written responses to non-routine problems revealed that many students directly applied memorized procedures without providing conceptual justification. - Interview data confirmed that students tended to rely on previously demonstrated solution steps and reported difficulties when required to construct independent reasoning. - A review of students' achievement records showed that 56.7% of students did not achieve a passing grade (< 60) on the circle topic.
2.	Student characteristics	- Interviews indicated that students experienced difficulties in understanding fundamental concepts of circles, particularly when connecting algebraic representations with geometric interpretations. - Although students were familiar with GeoGebra from prior coursework, interviews and observations suggested that they primarily used it for procedural visualization rather than exploratory reasoning.
3.	Supporting facilities	- Observational data confirmed the availability of computer laboratories with internet access to support technology-integrated instruction. - Students reported having personal mobile devices with stable internet connectivity.

Formative Evaluation Result

The designed Geo-Sheet teaching materials were validated by four experts, consisting of two material experts and two media experts. The results of the expert assessments were analyzed using Aiken's V formula (Retnawati, 2016). The Aiken's V values presented in Table 2 represent the average indices obtained from all validation items for each expert group.

Table 2. Summary of Content Validity Results Based on Aiken's V

No	Validator	Aiken's V	Category
1.	Material experts	0.89	Very Valid
2.	Media experts	0.85	Very Valid

The obtained Aiken's V coefficients in Table 2 indicate that all items met the minimum validity criterion (V ≥ 0.80), demonstrating that the Geo-Sheet teaching materials are valid in terms of alignment with teaching material development principles, content suitability, presentation (construction), and language. These findings confirm that the developed materials align with the intended learning objectives and are suitable for use in analytic geometry learning supported by GeoGebra.

Based on Table 2, the interactive Geo-Sheet teaching materials demonstrate a high level of content validity, as indicated by the descriptive interpretation of Aiken's V values. The validation results show that both the material and media aspects are considered valid and suitable for use, with minor revisions made in accordance with expert suggestions. These results are supported by previous studies (Nuryadi et al., 2020; Pertiwi et al., 2023; Shadaan & Kwan Eu, 2013) which developed GeoGebra applet-based learning materials on circle topics. The Geo-Sheet teaching materials are therefore considered to have appropriate content alignment with circle topics, as well as task and activity designs that support Creative Mathematical Reasoning.

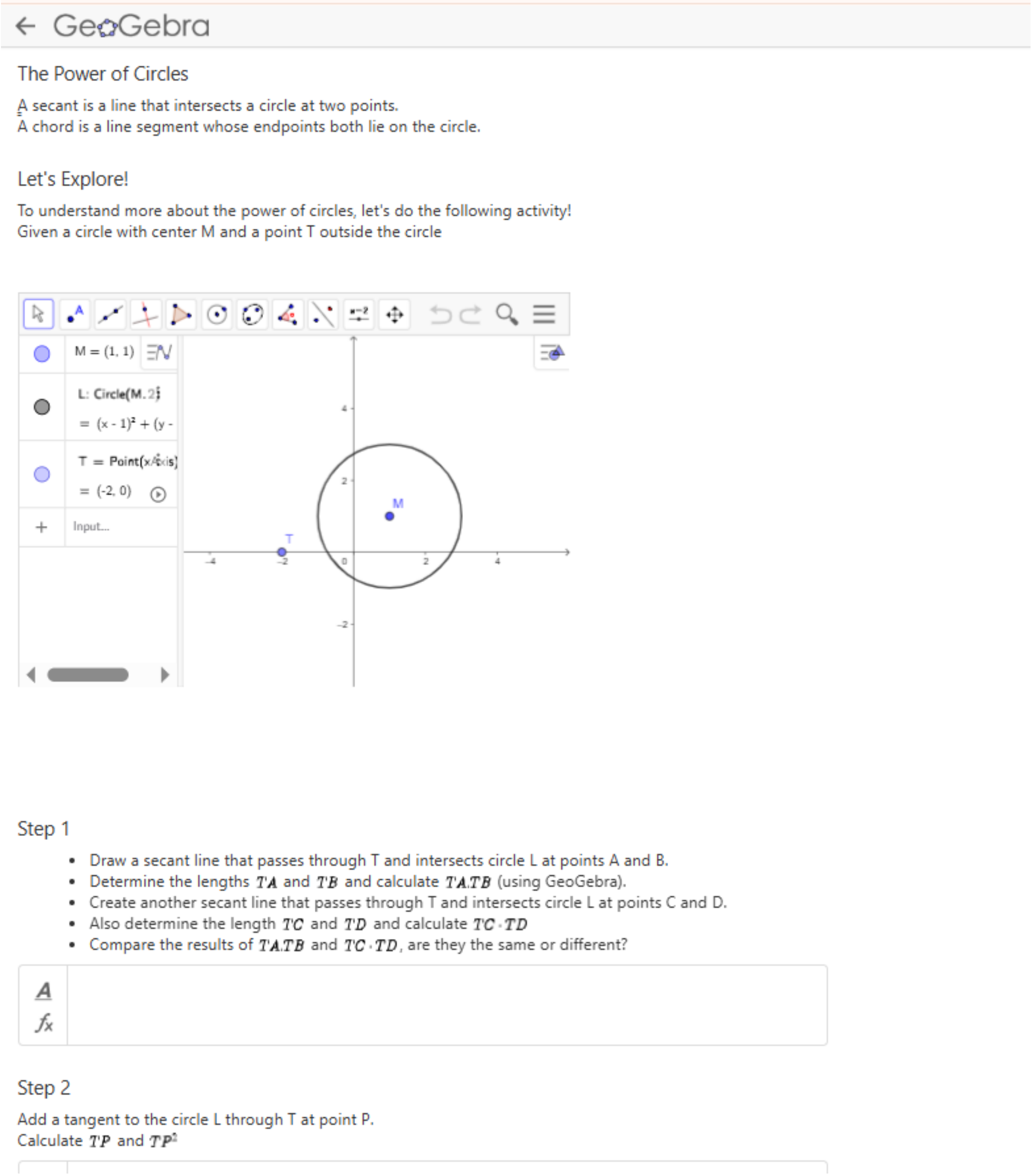
After content validity was confirmed through Aiken's V analysis, expert review and one-to-one trials were conducted in parallel to obtain qualitative feedback for product refinement. The feedback focused on usability, learning activities, and technical accessibility, and served as the basis for revising the Geo-Sheet teaching materials. A summary of expert suggestions and one-to-one trial results used for product revision is presented in Table 3.

Table 3. Summary of Expert Suggestions and One-to-One Trial Feedback

No	Suggestions	Revision
1.	In the exploration section related to differences in coefficient values in the general form of a circle equation, it is recommended to provide a GeoGebra template so that students can more easily observe emerging patterns.	Provide a single GeoGebra template accompanied by a set of guiding questions.
2.	Learning activities should, as much as possible, encourage students to construct their own understanding to foster Creative Mathematical Reasoning. Additional exploratory activities are recommended.	Add learning activities in Geo-Sheet that emphasize exploration and/or investigation, including the incorporation of new and varied question types beyond those in existing references.
3.	The GeoGebra feature in Geo-Sheet takes a considerable amount of time to load, particularly when accessed via smartphones.	Use smaller applet dimensions to optimize accessibility and usability on devices with limited screen sizes.

Based on input from experts and the one-to-one trial results presented in Table 3, the Geo-Sheet was revised. The researcher added exploration activities to stimulate students' thinking rather than simply applying previously learned algorithms (McJames et al., 2023). These exploration activities in Geo-Sheet enable students to discover the concept of circles interactively (Moreno & Mayer, 2007, Jonsson et al., 2020). The revised Geo-Sheet is called Prototype II.

Geo-Sheet includes an introduction to concepts supported by GeoGebra visualizations, exploration activities, worked examples, and interactive practice questions. The exploration activities are integrated with GeoGebra to support dynamic visualization. Figure 2 illustrates an exploration activity in Geo-Sheet designed to help students develop the concept of a secant, defined as a line that intersects a circle at two distinct points.



The Power of Circles

A secant is a line that intersects a circle at two points.
A chord is a line segment whose endpoints both lie on the circle.

Let's Explore!

To understand more about the power of circles, let's do the following activity!
Given a circle with center M and a point T outside the circle

M = (1, 1)
L: Circle(M, 2)
= (x - 1)² + (y -
T = Point(xAxis)
= (-2, 0)

Step 1

- Draw a secant line that passes through T and intersects circle L at points A and B.
- Determine the lengths TA and TB and calculate $TA.TB$ (using GeoGebra).
- Create another secant line that passes through T and intersects circle L at points C and D.
- Also determine the length TC and TD and calculate $TC \cdot TD$
- Compare the results of $TA.TB$ and $TC \cdot TD$, are they the same or different?

Step 2

Add a tangent to the circle L through T at point P.
Calculate TP and TP^2

Figure 2. Exploration Activities to Find the Concept of the Secant Line

The exploration activity in Figure 2 enables students to construct concepts prior to formal presentation through a guided discovery approach. In addition, Geo-Sheet incorporates exploration activities to support the construction of the circle power line concept. Typically, the equation of the circle power line is introduced to students as a ready-made formula. Although students may be able to determine the equation correctly, they often struggle to interpret the concept of the power line and how it is constructed.

In this activity, students are guided to develop knowledge structures to gain deeper conceptual understanding, which is a characteristic of tasks that foster Creative Mathematical Reasoning (Moreno & Mayer, 2007; Jonsson et al., 2020). Geo-Sheet is integrated with GeoGebra as an effective tool for developing students' mathematical thinking in analytic geometry (Khalil et al., 2019).

During the small-group test, the revised Geo-Sheet (Prototype II) was implemented with six students of varying ability levels. The Geo-Sheet link was provided to students (https://www.geogebra.org/classroom/phdakywd). Each student completed the exploration activities individually within Geo-Sheet. Students were able to input answers interactively for each question presented. After completing their independent exploration, students compared their results with peers and then generalized their findings to develop the concept of circles.

Following the use of Geo-Sheet, students provided feedback based on their experiences, including comments on readability and the ease or difficulties they encountered. Some suggestions from the small-group test are presented in Table 4.

Table 4. Suggestions from Small Group Test

No.	Suggestion	Revision
1.	The instructions or steps for using the Geo-Sheet are not sufficiently clear; therefore, guidance from the lecturer was required.	Clarify the instructions in the Geo-Sheet, especially regarding the use of GeoGebra.
2.	For students who are not familiar with the GeoGebra application, additional time is needed to adapt to its navigation and features. It is recommended to provide a short tutorial on how to use GeoGebra.	Add a short tutorial within the Geo-Sheet explaining the steps for using GeoGebra.
3.	When using the Geo-Sheet teaching materials, a stable internet connection is required.	Ensure a stable internet connection when using Geo-Sheet in learning activities (e.g., by using computer laboratories).

Based on the data in Table 4, the researcher revised the Geo-Sheet teaching materials. The revised Geo-Sheet, based on the small-group test results, is referred to as Prototype III. Prototype III was then implemented in one class as a field test involving 30 students.

After learning the circle material using Geo-Sheet, a post-test was administered, and students completed a questionnaire distributed via Google Forms. The practicality of Geo-Sheet was evaluated based on three aspects, namely effectiveness, efficiency, and creativity. The questionnaire data were analyzed following Azwar (2010), using the calculation of mean values and standard deviations. The teaching materials are considered practical if a minimum practicality score of 57 is achieved.

In the questionnaire responses (n = 30), 70% of students rated Geo-Sheet as very practical, 26.67% rated it as practical, and 3.33% rated it as fairly practical. No students rated it below the fairly practical category. The practicality analysis confirms that Geo-Sheet is practical, as reflected in the majority of student responses, as shown in Figure 3.

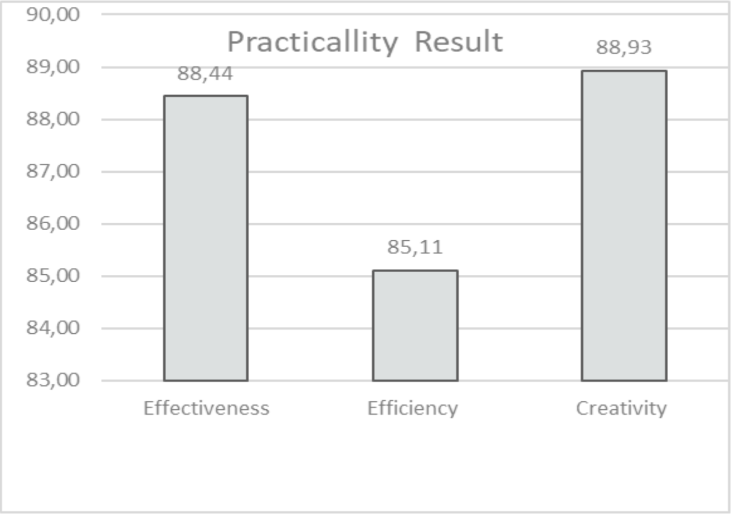


Figure 3. Results of Questionnaire Analysis Response

Based on Figure 3, the Geo-Sheet teaching materials received high ratings in the aspects of effectiveness, efficiency, and creativity, indicating an excellent level of practicality. The high scores for effectiveness and creativity suggest that Geo-Sheet not only helps students understand analytic geometry concepts through the integration of visual and symbolic representations, but also encourages them to explore various strategies, verify the correctness of their results, and construct solutions independently. Meanwhile, the efficiency score, which remains in the practical category, indicates that Geo-Sheet is easy to use and does not hinder the exploration process.

Table 5. Student Comments about Geo-Sheet

No	Student Response
1.	In my opinion, this GeoGebra-based teaching material makes learning easier because we can directly check the correctness of our answers using GeoGebra.
2.	It facilitates learning because students can explore and experiment, allowing them to visualize the material more clearly and realistically.
3.	Geo-Sheet makes learning easier because it enables students to verify and draw conclusions based on the steps they have taken.
4.	Geo-Sheet is helpful because students can immediately learn how to apply concepts and compare them with manual calculation processes.
5.	Geo-Sheet supports learning in analytic geometry, especially due to its visual and interactive nature. Students can experiment by changing the position of a point or the slope of a line and immediately observe the effects. However, limited mastery of GeoGebra may become a barrier for some students..
6	Geo-Sheet makes learning easier because errors are immediately visible and can be corrected.

Table 5 presents students' comments on the Geo-Sheet. The questionnaire results

These findings indicate that Geo-Sheet, integrated with GeoGebra, is perceived as practical and supportive of students' learning and creativity. Students reported that it helped them understand concepts, explore relationships, and remain engaged during circle learning activities. These findings align with previous studies showing that GeoGebra-based worksheets enhance students' interest, critical and creative thinking, problem-solving skills, and engagement in mathematics learning (Granberg & Olsson, 2015; Haryani & Hamidah, 2022; Hidayat et al., 2024; Selvy et al., 2020; Shadaan & Kwan Eu, 2013).

Potential Effect on CMR

The analysis of the potential effect was conducted descriptively by comparing pre-test and post-test mean scores across CMR indicators. The results from the pre-test and post-test of 30 students indicated an increase across all CMR indicators. The pre-test and post-test results are shown in Figure 4.

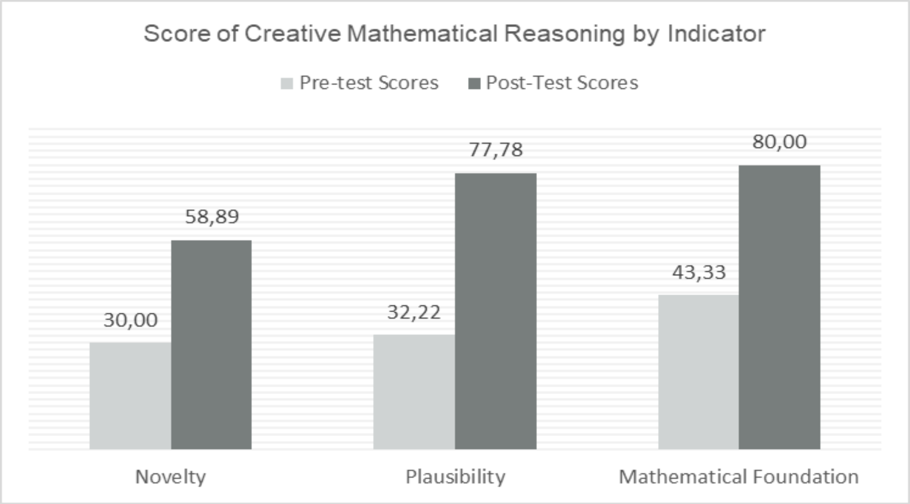


Figure 4. Pre-Test and Post-Test Scores of Students in the Field Test

Figure 4 shows that novelty increased by 28.89 points, plausibility by 45.56 points, and mathematical foundation by 36.67 points. These results suggest that Geo-Sheet not only facilitated procedural learning but also encouraged higher-order reasoning aligned with the Creative Mathematical Reasoning (Lithner, 2017).

Evidence of CMR Stimulation

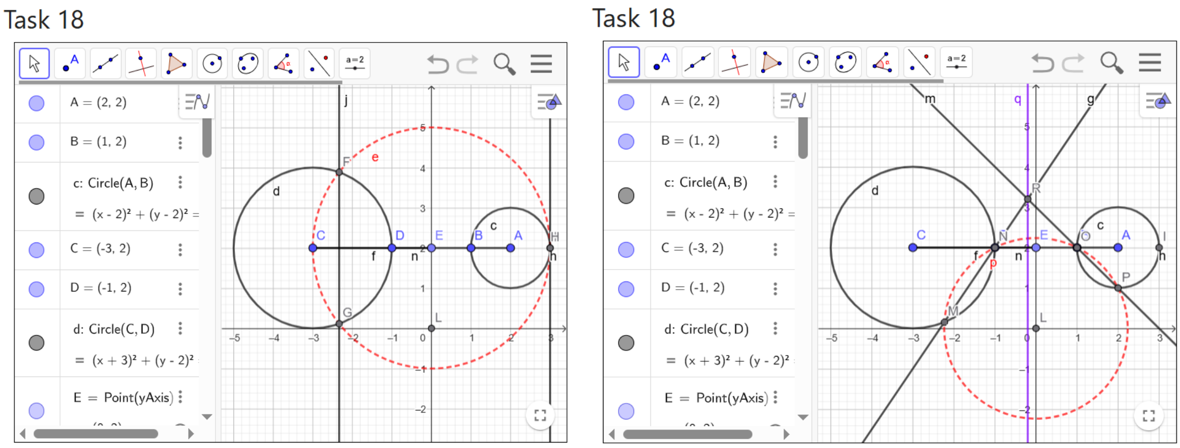
Two students (coded as DC and DP) were purposively selected for in-depth analysis because their solution processes clearly illustrated critical moments reflecting novelty, plausibility, and mathematical foundation during Geo-Sheet activities.

Case 1: Radical Axis Construction (Student DC)

Student DC was asked to construct the radical axis of two disjoint circles using Geo-

Sheet instructions. The task required students to follow three structured steps: (1) construct a third circle intersecting both given circles, (2) determine the radical axes of each pair of circles, and (3) draw the perpendicular line through the intersection point. However, the circles chosen by DC produced parallel tangents, so the radical axis could not be constructed visually. Instead of stopping, DC examined the situation analytically and obtained the equation $x = -0.2$. However, the student still could not visualize the line because the intersection point of the radical axis was not obtained. After guidance from the instructor, DC modified the construction by shifting one circle so that the centers were no longer collinear. This adjustment allowed the radical axis to be constructed correctly.

R : *Can you draw the radical axis?*
DC : *Not yet. I cannot find the intersection points of each pair of radical axes, so I cannot draw the radical axis*
R : *Have you tried it analytically? What is the equation of the radical axis? (DC finds the equation of the radical axis using the formula.)*
DC : *After I calculated using the formula, it turns out that the radical axis of the two circles is x = -0.2. However, I still cannot construct the graph because I cannot find the intersection point of the radical axis.*
R : *Try shifting the constructed circle so that its center is not collinear with the others, then observe what happens*
DC : *After I shifted it, the radical axis intersected, and the line I wanted could be constructed.*


Task 18

A = (2, 2)
B = (1, 2)
c: Circle(A, B)
= (x - 2)² + (y - 2)² =
C = (-3, 2)
D = (-1, 2)
d: Circle(C, D)
= (x + 3)² + (y - 2)²
E = Point(yAxis)

Task 18

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= (x + 3)² + (y - 2)²
E = Point(yAxis)

Figure 5. Construction the radical axis by DC before and after shifting the third circle

The transition from the unsuccessful to the revised construction is illustrated in Figure 5. This episode highlights how Geo-Sheet stimulated Creative Mathematical Reasoning (CMR). First, novelty emerged when DC departed from the given instructions and constructed an alternative approach by shifting the circle's position. Although the task was guided, the breakdown in visualization created a problem situation that required the student to generate a new solution path. Second, plausibility was demonstrated by verifying the radical axis analytically and confirming that a valid equation ($x = -0.2$), could be obtained. Finally, mathematical foundation was evident in the use of circle equations and properties of radical axes to justify the reasoning. This situation is consistent with Lithner (2017) framework, which emphasizes that creative reasoning emerges when students encounter problem situations that cannot be solved using memorized procedures.

The radical axis task initially failed due to the collinearity of the circle centers, forcing DC to move beyond imitative reasoning. By modifying the construction (shifting the circle), DC engaged in novelty- and plausibility-based reasoning grounded in mathematical principles. This also aligns with Jonsson et al. (2020) who argue that well-designed tasks guide students to attend to structural features—in this case, the relative positions of circle centers. Moreover, the guided instructions in Geo-Sheet provided scaffolding (Moreno & Mayer, 2007), while the interactive dialogue created opportunities for exploration and discussion that are central to fostering mathematical reasoning (Lithner, 2017; Olsson & Granberg, 2024). These findings indicate that Geo-Sheet functions most effectively when complemented by teacher support and instructional orchestration, particularly in guiding mathematical discussion and aligning students' exploratory activities with intended learning goals.

Case 2: Polar Line Construction (Student DP)

Student DP was tasked with finding the polar line of a point P with respect to a circle. After manually calculating the equation, DP verified the result using GeoGebra by constructing tangents from point P and identifying their points of tangency. DP confirmed that the resulting line was indeed the polar line. When prompted to use GeoGebra's built-in "polar" tool, DP observed that the algebraic equation produced differed from the manually obtained result, although the visual representation was identical. The following is interview data with participant DP:

P : *Is it correct that this is the equation of the required line?*
DP : ... (DP verifies the answer by drawing two tangent lines to the circle through point P and identifying their points of tangency.) *That's correct. The line I constructed passes through the intersection point of the two tangents to the circle drawn from point P. This means that the line I found is a polar line.*
P : *Please check it directly in GeoGebra. Which tool should you use?*
DP : *It seems to be the "polar" tool.* (DP constructs the polar line using the tool.)
P : *What is the result?*
DP : *The polar line equation from my calculation is different from the GeoGebra result, but the figure is the same.*

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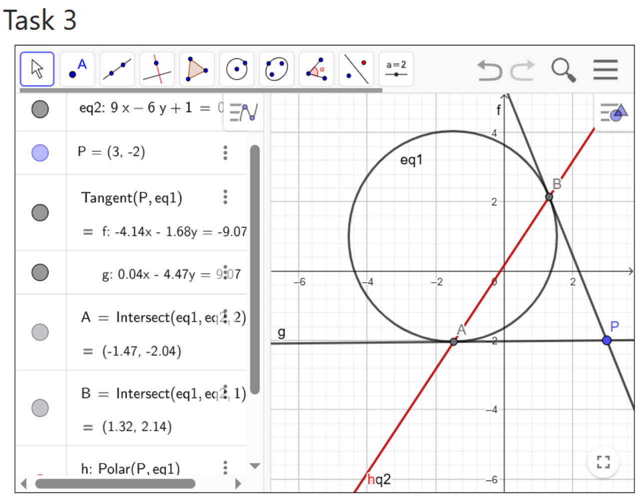
Task 3

eq2: 9 x − 6 y + 1 = 0
P = (3, -2)
Tangent(P, eq1)
= f: -4.14x - 1.68y = -9.07
g: 0.04x - 4.47y = 9.07
A = Intersect(eq1, eq2, 2)
= (-1.47, -2.04)
B = Intersect(eq1, eq2, 1)
= (1.32, 2.14)
h: Polar(P, eq1)

Figure 6. DP Makes Polar Line Construction to Verify Manual Count

The comparison between the manually derived polar line and the GeoGebra-generated construction is presented in Figure 6. The situation in Figure 6 stimulated the following reasoning: first, novelty appeared when DP combined two approaches—manual calculation and software-based verification—to test the result. Plausibility emerged when DP reasoned that, although the symbolic forms were different, both represented the same line geometrically. Finally, mathematical foundation was evident when DP connected algebraic manipulation with the geometric properties of the polar line.

The episode with Student DP further illustrates how Geo-Sheet tasks can stimulate CMR. When DP compared the manually calculated polar line with the construction using GeoGebra's polar tool, the two equations differed even though the visual representations were identical. This situation created a moment of uncertainty and required DP to reconcile symbolic and visual reasoning. According to Lithner (2017) framework, such moments exemplify creative reasoning, since the task cannot be solved by merely recalling memorized procedures but instead requires critical evaluation of multiple solution paths. Similarly, Jonsson et al. (2020) emphasize that CMR tasks should direct learners' attention to structural features essential for understanding—in this case, recognizing that different algebraic forms can represent the same geometric object. The use of guided but open-ended tasks in Geo-Sheet reflects the principle of guided inquiry (Moreno & Mayer, 2007), which supports students in selecting, organizing, and integrating information. Moreover, the student's active checking, hypothesis testing, and use of dynamic tools illustrate the exploratory aspects highlighted by Granberg and Olsson (2015). Thus, this case demonstrates how Geo-Sheet integrates visualization and symbolic reasoning to foster novelty, plausibility, and mathematical foundation in students' reasoning.

Synthesis of Cases

Both episodes demonstrate how Geo-Sheet stimulated students to go beyond imitative reasoning. Breakdowns in visualization (Case 1) and differences in algebraic representations (Case 2) became problem situations that encouraged students to problematize, verify, and reconstruct solutions. Geo-Sheet thus functioned not only as a visualization tool but also as a task design that systematically stimulated novelty, plausibility, and mathematical foundation in students' reasoning.

Theoretical Implications

The findings support Lithner (2017) view that CMR emerges when learners face unfamiliar or problematic situations that cannot be solved through rote procedures. Geo-Sheet creates such situations by combining guided instructions with opportunities for exploration, enabling students to construct their own reasoning paths. Its design is grounded in guided inquiry theory, where learners actively select, organize, and integrate new information, thereby fostering generative cognitive processes (Moreno & Mayer, 2007).

Within this framework, the exploratory tasks in Geo-Sheet are intentionally designed to support collaboration, hypothesis testing, and discussion as complementary components of the learning process. Collaboration is fostered through shared exploration, where students manipulate dynamic objects, exchange ideas, and negotiate alternative solution paths. Hypothesis testing is enabled by the immediate feedback provided by GeoGebra, allowing students to formulate conjectures, test them dynamically, and revise their reasoning based on observed outcomes. Discussion is embedded through prompts that require students to explain, justify, and evaluate both their own and peers' conjectures using visual and mathematical evidence. Together, these processes stimulate curiosity, encourage multiple solution strategies, and nurture creative ideas that strengthen mathematical reasoning (Granberg & Olsson, 2015; Vale et al., 2017).

The novelty of this study lies in integrating dynamic visualization with guided exploratory tasks to systematically stimulate the three dimensions of CMR: novelty, plausibility, and mathematical foundation. Previous studies on GeoGebra have mainly highlighted its role in visualization (Nuryadi et al., 2020; Yildiz et al., 2017), whereas this study demonstrates how Geo-Sheet can function as a pedagogical design model that connects visualization, inquiry, and reasoning. Thus, Geo-Sheet extends the theoretical discussion of technology-assisted learning from procedural and visual support toward the development of creative mathematical reasoning.

Across cases and theoretical analysis, a consistent design mechanism can be identified. Geo-Sheet combines dynamic visualization with structured exploratory prompts that deliberately create productive problem situations. The GeoGebra environment enables students to manipulate objects and observe invariant relationships, while the embedded prompts require them to justify, compare, and verify their reasoning analytically. When visual outcomes conflict with symbolic results, or when constructions initially fail, students are encouraged to reformulate conjectures and refine their arguments. This interaction between manipulation, feedback, and guided reflection appears to support the emergence of novelty, plausibility, and mathematical foundation in students' reasoning.

Limitations and Future Research

The study is limited to a single institution and one geometry topic (circles). Further research should apply Geo-Sheet to other mathematical topics and different educational contexts to test its robustness and generalizability. Comparative studies may also explore how Geo-Sheet performs relative to other instructional innovations to deepen understanding of how technology can systematically foster CMR.

CONCLUSION

This study developed and evaluated Geo-Sheet as a GeoGebra-integrated teaching material for analytic geometry, focusing on expert validity, student practicality, and its potential effect on supporting CMR. The results indicate that Geo-Sheet demonstrates high validity, while its use in instruction shows high practicality and promising potential in supporting students' CMR development.

The qualitative analysis reveals that Geo-Sheet supports CMR through the integration of dynamic visualization and guided exploratory prompts. By encouraging students to formulate conjectures, test them through manipulation, and reconcile visual and symbolic representations, the design creates productive problem situations that stimulate novelty, plausibility, and mathematical foundation. However, the effectiveness of these processes depends on teacher support and instructional orchestration, particularly in facilitating discussion, monitoring conjecture development, and maintaining mathematical focus during exploration.

Therefore, Geo-Sheet should be understood not as a stand-alone instructional tool but as a pedagogical design that requires careful classroom implementation. This study is limited to one mathematical topic and institutional context. Future research may investigate how design features and teacher orchestration interact across different topics and learning environments to further strengthen technology-supported CMR development.

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