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From complex to simple: Analyzing prospective teachers' analogy reasoning in creating accessible geometry problems

Abdul Haris Rosyidi[1], Cholis Sa'dijah[2], Heri Purnomo[3], and Abdul Halim Abdullah[4]

[1,3]Department of Mathematics Education, Universitas Negeri Surabaya, Surabaya, Indonesia
[2] Department of Mathematics Education, Universitas Negeri Malang, Malang, Indonesia
[4]School of Education, Universiti Teknologi Malaysia, Malaysia

ABSTRACT

Students often struggle to identify relevant analog problems when solving new tasks, highlighting the need for teachers to design simple analog problems that serve as scaffolding. This study aims to analyze prospective teachers' analogical reasoning processes in simplifying complex geometry problems using the Analogical Reasoning in Mathematics (ARM) framework. This qualitative research involved 34 prospective mathematics teachers from a public university in Surabaya, Indonesia. Participants were selected through purposive sampling based on their academic performance and prior coursework in geometry and problem solving. Data were collected through task-based interviews, written work, and observations during problem-simplification activities. The collected data were analyzed thematically, guided by the components of the ARM framework. The results indicate that prospective teachers with varying ability levels employed different analogical reasoning strategies to simplify complex problems through ARM activities. High-ability prospective teachers identified a broader range of student difficulties and adapted the problems into two-step analog problems featuring variations in visual representations, number of circles, and geometric shapes. Conversely, low-ability prospective teachers focused on difficulties related to verbal representation and the need for concrete numerical information, adapting the problems into single, highly simplified analog problems with specific images and numbers. Overall, prospective teachers actively utilized analogical reasoning to design analog problems that addressed student difficulties. Differences in ability were associated with the complexity of adaptation strategies and the depth of difficulty identification, underscoring the importance of training prospective teachers to integrate both approaches to effectively support student understanding.

KEYWORDS

Analogy reasoning; complex problems; simple problem; geometry problem; prospective teachers

CORRESPONDING AUTHOR

Abdul Haris Rosyidi ✉ abdulharis@unesa.ac.id Department of Mathematics Education, Universitas Negeri Surabaya, Surabaya, Indonesia

INTRODUCTION

Most researchers agree that a problem refers to a task that an individual must solve but does not immediately know how to approach (Lester, 2013; Kirisci et al., 2020; Riegel, 2021). This definition highlights a gap between the goal of completing a task and the absence of an algorithm that can be directly applied to solve it. A task becomes a problem for an individual when they clearly understand the task, know the expected goal, have the necessary resources, and are able to use those resources effectively (Moursund, 2005; Lupiáñez et al., 2024). Polya (1957) emphasized several questions that teachers can pose to guide students in problem-solving, many of which are related to analogical reasoning. This suggests that finding an appropriate, similar, or analog problem can assist in solving a given problem.The process of

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identifying a simple analog problem is not easy for students. Richland et al. (2004) & Zhu et al. (2024) noted that the ability to distinguish among several source problems that are analogical and relevant when solving new problems is crucial—yet students often struggle with this. This indicates that teachers need to prepare simpler analog problems as scaffolds in problem-solving instruction. In the context of problem analogy, the simple analog problem serves as the *source problem*, while the given problem is the *target problem*. Therefore, teachers must design a variety of potential source problems to assist students in addressing target problems.

Constructing source problems involves simplifying the problems that students will solve, producing outcomes that can help them when they encounter difficulties during problem-solving. To simplify problems effectively, teachers must thoroughly understand the structure of the problem while considering students' prior knowledge and potential difficulties as problem solvers (Woods & Copur-Gencturk, 2024). Teachers also need sufficient knowledge of problem-posing practices (Woods & Copur-Gencturk, 2024) since designing effective problems, especially analog problems, requires the intentional selection of contexts, mathematical structures, and difficulty levels that align with students' prior knowledge. Without this expertise, posed problems may be poorly structured, trivial, or disconnected from intended learning goals, limiting their usefulness in scaffolding problem solving.

Knowledge of problem-posing practices enables teachers to anticipate potential student strategies and misconceptions, preserve the deep structure of the original task while adjusting surface features, and ensure that posed problems remain mathematically rich and solvable. With sound problem planning, teachers can provide appropriate scaffolds to help students overcome impasses during problem-solving. According to Vygotsky and Cole (1978), such assistance can be provided by offering relevant problems, and Polya (1957) noted that these relevant problems are often analog problems that students have previously solved.

Two challenges arise in using analogy—particularly problem analogy—in problem-solving instruction: (1) the source problems are often unfamiliar to students, and (2) the relational similarity between the source and target problems is clear to the teacher but not to the students (Clement, 1991; Kroczek et al., 2022). These challenges are closely related to teachers' analogical reasoning when constructing source problems during lesson planning. Problem simplification, as an important aspect of problem-solving, helps identify the core structure of a problem (Branca, 1980; Calabrese et al., 2024). Polya (1957) stated that problem simplification aims to make a given problem easier to understand and ultimately to solve. Simplifying problems enables individuals to better understand and identify appropriate problem-solving

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strategies (Schoenfeld, 2014), and to focus on essential components while developing step-by-step solutions (Lester, 1980).

The process of identifying or constructing simple analog problems is indeed challenging for students. Although crucial in problem-solving, many students are not yet able to perform this process effectively (Richland et al., 2004). This reality underscores the need for teachers to prepare simple analog problems during lesson planning as a contingency for when students struggle to find solutions. In the context of geometry, research has shown that teaching two-dimensional geometry is particularly challenging (Yi et al., 2020), Students' performance in geometry and measurement remains low (Steele, 2013; Danlami et al., 2025), and the geometric thinking levels of secondary students are often still limited (Susanto & Mahmudi, 2021; Amalliyah et al., 2021).

The activity of preparing simple analog problems is closely tied to analogical reasoning, as the structure of the created problems must inspire students to find solutions to the original problem. The analogical reasoning involved in this process can be described as *analogical reasoning in problem simplification*. Previous studies (e.g., Kristayulita et al., 2020; Dai et al., 2023; Hasan et al., 2024) have identified differences in the analogical reasoning employed by prospective teachers when simplifying problems.

This study addresses the issue of prospective mathematics teachers' limited ability to construct effective simple analog problems. These limitations often stem from challenges in analogical reasoning and problem simplification. Given the crucial role of analogical reasoning in breaking down complex problems, particularly in geometry, into simpler forms that enhance student understanding, this research aims to explore how prospective teachers engage in such reasoning when simplifying geometry problems.

The investigation is guided by the Analogical Reasoning in Mathematics (ARM) framework (Hicks, 2024), which was developed to understand how learners use analogical reasoning in mathematics. The ARM framework views analogy as reasoning about underlying structural or relational similarities between two domains, involving the mapping of objects and their relationships from a source domain to a target domain. By breaking down reasoning into individual analogy activities, the framework allows detailed analysis of thinking processes, even when learners produce similar final answers. ARM identifies several aspects of analogical reasoning, including reasoning within a single domain, recognizing differences across domains, and focusing on particular domains. Its general pathway consists of three main stages: *Analogous Access*, where learners recall relevant concepts from the source domain; *Analogy*

Generation, where they construct the analogy and; *Establishing New Content*, where new mathematical ideas or structures emerge in the target domain as a result of the analogy process.By applying the ARM framework, this study aims to provide both theoretical and practical contributions to mathematics education, particularly within teacher preparation programs. The findings are expected to inform training approaches that enhance prospective teachers' skills in designing meaningful analog problems, thereby improving their ability to scaffold students' problem-solving processes effectively.

METHODS

This study employed qualitative research design using a case study approach (Sin & Lin, 2025). The subjects were prospective teachers enrolled in the Mathematics Education program at a public university in Surabaya, Indonesia. Participants were selected from a pool of 34 prospective teachers who constructed simple analog problems demonstrated consistency in the structural relationships between the source and target problems.

Data were collected through task-based interviews in which participants were presented with geometry problems to solve. Their written solutions were gathered and followed by semi-structured interviews designed to explore their reasoning and thought processes related to analogical problem simplification (Takona, 2024). The responses obtained from the task-based problem-solving activities were initially categorized based on variations in the simple analog problems they produced, specifically focusing on the consistency between the structures of the source and target problems. This categorization was based on participants' written work.

From each group of consistent responses, one representative was purposively selected for a follow-up interview, with additional consideration given to the participant's communicative clarity. Data from both the written problem-solving tasks and the semi-structured interviews were analyzed using thematic analysis, following the stages of data reduction, data display, and conclusion drawing (Miles & Huberman, 2014). The analysis of prospective mathematics teachers' analogical reasoning in simplifying geometry problems was guided by the Analogical Reasoning in Mathematics (ARM) framework (Hicks, 2024).

RESULTS AND DISCUSSION

Using the Analogical Reasoning in Mathematics (ARM) framework, the subject's analogical reasoning activities in simplifying complex problems are elaborated. The complex problem involves comparing the total area of three circles with equal radii inside a triangle to the total area of the circular regions located outside the triangle. Prospective mathematics

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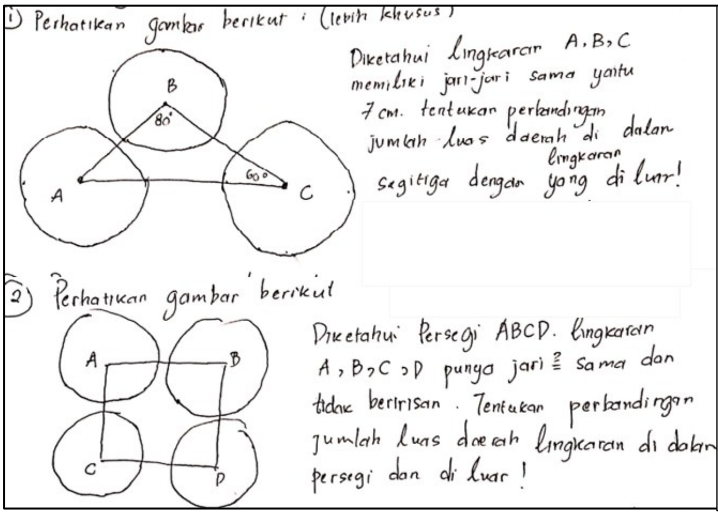
teachers' analogical reasoning is distinguished between high-ability prospective mathematics teachers (MS) and low-ability prospective mathematics teachers (FS), categorized based on their cumulative grade point average (GPA). The analogical reasoning activities involved in simplifying the complex problem are outlined as follows.

Results

High-Ability Prospective Mathematics Teachers' (MS) Analogical Reasoning Activities

The analysis of the MS subjects' simplification of complex problems focuses on comparing the total area of three circles with equal radii inside a triangle to the total area of the circular regions outside the triangle. This complex problem, which involves the relationship between the sector areas of the circles and the triangle's angles, requires an understanding of geometric relationships and proportional reasoning.

To make the problem more accessible for middle school students, the MS subjects created two simpler analog or mirrored problems involving either one or four circles combined with a triangle or a square. These problems were intentionally designed to maintain the structural relationship of the original problem while reducing its complexity. Figure 1 presents the simpler problems created by the MS subject, as shown below.



1. Consider the following figure. Circles A, B, and C each have a radius of 7 cm. Determine the ratio of the combined area of the portions of the circles that lie inside the triangle to the combined area of the portions of the circles that lie outside the triangle.

2. Consider the following figure. ABCD is a square. Circles A, B, C, and D each have the same radius and do not overlap. Determine the ratio of the combined area of the portions of the circles that lie inside the square to the combined area of the portions of the circles that lie outside the square.

Figure 1. Simple problems created by the MS subject

In producing the simple problems above, the researcher also conducted an interview with subject MS, as follows.

R : When middle school students are faced with these problems, where do you think the difficulties lie?

MS : The difficulty for junior high school students is that the problem is presented directly as a story without a picture. There is also the possibility of making mistakes when constructing a representation. Moreover, the problem is not one they usually encounter

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at school.

MS : Perhaps they already have a pictorial representation, but it is still difficult for them to find the relationship between the concept of the sector area (juring) and the sum of the angles in the triangle to solve the problem.

R : This question is verbal and has no accompanying image. Did you consider presenting the question verbally or directly with pictures?

MS : In my opinion, one of the difficulties students experience arises from the absence of a representation, so they have to think twice. To create analog problems, I minimize students' difficulties without losing the learning goal. Therefore, I designed analog problems that already include pictures.

R : Here are the two problems you created (pointing to the figure). To reach the complex problem, do you need to create both, or would one be enough?

MS : Initially, I thought the first problem was enough. But after reconsidering, I thought of another problem that could further guide students. In the first problem, the angle measures are known, while in the second, they are unknown but more familiar to students, and the concept of angle measures in a square is easier to understand.

R : What is the focus of your first question?

MS : To help students recall that the sum of the angles in a triangle is 180 degrees. One of the angles is unknown, so students must determine its measure to understand the relationship between the total angle sum of 180 degrees and the area of the sector.

R : And what is the purpose of your second question?

MS : The main problem is that the angle is unknown. I designed the second question to show that even if the angle size is not given, the problem can still be solved because the total angle in a square is 360 degrees. Actually, the second analog problem could also use an equilateral triangle, but I chose a square because it is more familiar to students and reduces possible errors. Since all right angles in a square add up to 360 degrees, it is easier for them to understand.

R : For the second question, why did you choose four circles instead of one, two, or three?

MS : It's the same concept—the main focus is on the total angle relationship, not the number of circles.

R : So, what is the analogy between your problems and the complex problem?

MS **:** *The first analogy connects the concept of the area of the circle's sector with the angle measure in a triangle. The second focuses on situations where the angle measure is unknown.*

Based on Hicks' *Analogical Reasoning in Mathematics (ARM)* framework (Hicks, 2024), the analogical reasoning activities of subject MS are described below.

Adapting

MS made several adaptations to the analog problems to simplify the original one. She decided to include diagrams so that students would not struggle with verbal representations in the complex problem, "*If there's no picture, they have trouble imagining it, so I added a circle diagram, so the sector is visible.*"

This adaptation aimed to address representation difficulties. MS also changed the number of circles—from three in the complex problem to one in the first analog problem and four in the second, "*It's easier that way, so they won't get confused by too many circles. With* *just one, they can focus on a single sector first.*"

Additionally, MS used a triangle with an unknown angle in the first analog problem to elicit the application of the triangle angle sum (180°), and a square in the second to highlight the property of 90° angles, "*I changed the shape to a square—its angles are already 90°, so it's easier for them.*"

Associating

MS linked the total angles in a geometric shape (triangle or square) with the total area of a circle or its relevant portion, "*The total angle in a triangle or square is like the total of a full circle, so the proportion can be compared*". This demonstrates how MS aimed to help students understand the relationship between the area of a circle's sector and the total angle of the polygon, associating the total angles of the shape with the full or partial circle area.

Distinguishing

MS clearly identified the aspects that would be difficult for junior high school students, "*If there's no picture, they find it hard to imagine. And if the angle isn't given, they're confused about where to start.*". She distinguished between challenges related to verbal representation without a diagram, the absence of a specific angle measure, and the unfamiliarity of such problems in school settings. This process reflects the *distinguishing* stage of analogical reasoning—identifying the differences between students' potential knowledge and the demands of the source problem.

Exporting

MS retained the core relationship between the area of a sector and the total angles of a polygon from the original to the analog problems, "*The concept of comparing a circle's area with its angle I still kept, but I changed the shape to make it more familiar*". She preserved the essential conceptual link between the circle's sector area and the polygon's angle measures (triangle → square), transferring the relational structure from the source to the analog problems.

Recalling

MS recalled their own learning experiences and the types of circle problems commonly encountered in junior high school, "*I remember in junior high, most problems involved circles with given angles, so I made it similar to that*". This shows that prior learning experiences influenced how MS designed the analog problems. MS also recalled their own understanding of how to solve complex problems, linking them to the concepts of the area of a circle's sector and the sum of the angles in a triangle. They also referred to the types of circle area problems typically encountered by junior high school students, which usually involved one circle with known angles. Recalling prior knowledge of these mathematical concepts and common student experiences informed MS's approach in constructing simpler analogies.

The characteristics of MS's analogy construction reveal a **student-centered approach** focused on identifying specific difficulties, simplifying problems by isolating key concepts, and using familiar visual representations. MS deliberately maintained the structural similarity between the analog and complex problems to make the transfer of understanding easier, "*If the problem looks similar, they can more easily connect the method*". MS articulated analogies between their simple and complex problems, highlighting the relationship between the area of a circle's sector (*juring*) and the total angles in a triangle, as well as the notion of unknown angle measures. This reflected MS's awareness of the structural similarities they wanted to create and leverage to support student understanding.

In summary, MS engaged in several analogical reasoning activities guided by their understanding of the difficulties that junior high school students might face with complex problems. They differentiated challenges, recalled relevant mathematical knowledge and student experiences, exported core conceptual relationships, and significantly adapted visual representations, the number of circles, and the enclosing geometric shapes to create more accessible analog problems.

The most prominent characteristic of MS's analogy construction was their emphasis on identifying specific difficulties that junior high school students might face in solving complex problems. MS did not view the problem solely from the perspective of a prospective teacher who already understood the concepts but instead positioned themselves as a student to anticipate which parts of the problem could become obstacles to understanding. MS simplified complex problems by isolating key concepts that students might find difficult and focused the analog problems on these aspects. For example, in the first problem, MS broke down the difficulty into two parts—understanding the area of a circle's sector and the absence of given angle measures—and then created analog problems that separately highlighted each aspect. In the second problem, the main focus of MS's analog problem was on the concept of the height of an obtuse triangle.

MS also recognized the important role of visual representations in helping students understand the problem. In the first problem, which was originally presented verbally, MS created analog problems with accompanying pictures to support visualization. For the second problem, which was already pictorial, MS maintained the use of images to reinforce

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understanding of the analog concept. MS also tended to maintain the question structure between the analog and complex problems—especially in terms of asking for area comparisons—so that students could more easily recognize the relationship between the two problems and see how the solution to the analog problem could be applied to the complex one.

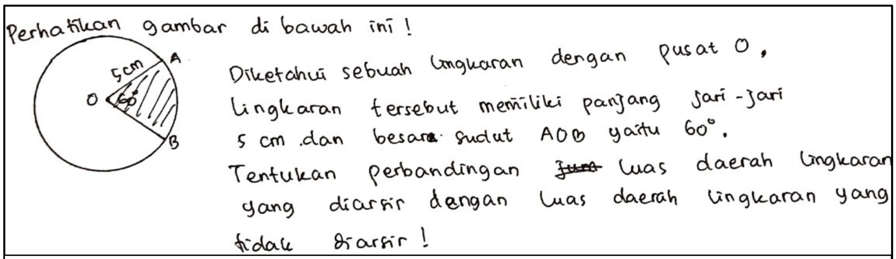
MS also demonstrated awareness that sometimes more than one analog problem is needed to gradually guide students toward understanding a complex concept. They realized that students' difficulties may vary, and a single analog problem may not be sufficient to bridge all gaps in understanding.

Overall, the characteristics of MS's analogy construction indicate a student-centered approach emphasizing the identification and analysis of student difficulties, simplification through focus on key concepts, and the use of familiar visual and structural representations to facilitate the transfer of understanding from analog problems to complex problems.

Lower ability (FS) prospective mathematics teachers' analogical reasoning activities

The complex problem presents three circles of equal radius centered at the vertices of a triangle, with the circles not intersecting each other. The task is to determine the ratio between the total area of the parts of the circles that lie inside the triangle and the total area of the parts that lie outside the triangle. This problem is presented verbally, without any accompanying figure.

The simple analog problem created by FS involves a single circle with a radius of 10 cm and a central angle of 60 degrees. The question asks for the ratio between the area of the shaded sector of the circle and the unshaded region (outside the sector). The simple problem constructed by FS is shown in Figure 2.



Consider a circle with center O and a radius of 5 cm. The sector AOB, with a central angle of 60°, is shaded. Determine the ratio of the shaded area to the unshaded area of the circle.

Figure 2. Simple problems created by the FS subject

In producing the simple problems above, the researcher also conducted an interview with subject FS as follows:

R : The complex problem involves two figures—a triangle and a circle. Now, your problem

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only involves a circle. What were your considerations for removing the triangle and focusing only on the circle?

FS : Because even without the triangle, students can still work on the circle... the key structure of this complex problem actually lies in the circle, so the triangle was eliminated.

FS : Actually, when I made this problem, I made up the angles and radii... because students usually struggle when there are no given numbers—it's difficult for them to work it out.

R : So, my question is presented in text form. Why did you decide to include a picture in your version?

FS : Because if there's no picture, students are usually confused... it's to inspire them that solving complex problems can start by sketching the picture first.

R : What concepts are involved in solving the problem, according to your understanding?

FS : The sum of the angles in a triangle and the area of a portion of a circle.

FS : If students already know how to find the area of a part of a circle and compare it with other areas, they can probably solve this problem.

FS : The triangle becomes irrelevant when the focus is on the circle... once students know the radius, they won't focus on the triangle anymore.

FS : At first, I didn't include the word "comparison"... but then I thought, if students don't know what comparison means, they won't be able to solve the problem. That's why I added that question.

R : Do you think 8th-grade junior high school students would find that difficult?

FS : The first difficulty is translating the problem into a picture... then determining the area of the circle when the radius isn't given... and identifying the position of the three circles because the triangle type and side lengths aren't specified—students usually get confused about that.

Based on Hicks' *Analogical Reasoning in Mathematics* (ARM) framework (Hicks, 2024), the analogical reasoning activity of subject FS is described as follows.

Adapting

FS simplified the complex problem by reducing the number of circles from three to one, removing the triangle, and adding numerical information (radius = 10 cm and central angle = 60°) to make calculations easier for students, "*Because even if the students don't involve the triangle, they can still work on the circle... the key structure of this complex problem is actually in the circle, so the triangle is eliminated*".

FS explained that the numerical values were not structurally necessary but were included to reduce students' difficulties in working without given data, "*Actually, when I made this problem, the angles and radius I made up... because students find it difficult to work without numbers*". Additionally, FS used visual representation to help students better understand the problem, unlike the original complex problem, which was presented entirely in text, "*Because if you don't use the picture, students are usually confused... it's to encourage them that solving complex problems can begin by sketching the picture first*".

Associating

FS associated the concepts of *sector area* and *area comparison* as the key mathematical ideas students need to solve the problem, "*Sum of angles in a triangle, area of a portion of a circle*". In the analog problem, FS placed greater emphasis on calculating the sector area and comparing it to the rest of the circle. FS expected that if students mastered this simpler comparison, they could transfer that understanding to more complex problems, "*If students already know how to find the area of a part of the circle and compare it with other parts, they can probably solve this problem*".

Although FS mentioned the triangle's angles as part of the original problem, the focus of the analogy shifted toward understanding curved area and comparison as foundational concepts. Thus, FS's analogy construction focused on **drastic simplification**—removing the triangle, providing concrete numerical data, and reducing representational complexity. The goal was to address students' difficulties with representation, non-numerical calculations, and comparison concepts, using a simpler context as a stepping stone toward understanding more complex problems.

Distinguishing

FS recognized the structural differences between the complex problem (a circle and triangle, presented verbally) and the analog problem (a single circle, presented visually), "*The triangle will be invisible when the focus is on the circle... once students know the radius, they won't pay attention to the triangle because their focus is on the circle*".

Exporting

FS transferred the core idea of *area comparison* from the complex problem to the analog problem. Initially, FS did not include the term *comparison* in the question but later added it to help students understand the intended concept, "*At first, I didn't use comparison... then I thought that if students don't know what comparison means, they won't be able to solve the problem. That's why I added the question*".

Recalling

FS recalled common difficulties faced by Grade 8 students, such as translating verbal problems into diagrams, determining areas without given measurements, and identifying the position of circles when triangle information is missing, "*The first difficulty is translating the problem into a picture... then determining the area of the circle when the radius is unknown... and identifying the location of the three circles based on the type of triangle and side lengths—they usually get confused about that*". FS also considered students' struggles in understanding the concept of comparison.

Based on the ARM framework, FS demonstrated a strong tendency to simplify complex problems by reducing cognitive load and adding concrete numerical data to make calculations more accessible. In the *adapting* phase, FS strategically removed non-essential elements (the triangle), retained the core structure involving the circle, and incorporated visual representation to support understanding. In *associating*, FS connected relevant mathematical concepts—particularly *sector area* and *area comparison*—as transferable ideas for future learning. FS showed the ability to *distinguish* structural differences between the original and analog problems, understanding how changes in form and representation shift students' focus. In *exporting*, FS explicitly integrated the core concept of *comparison* into the analog task for conceptual clarity. Finally, through *recalling*, FS demonstrated awareness of common student difficulties, such as visualizing problems, working without measurements, and managing multi-shape configurations.

This combination of simplification, conceptual focus, structural awareness, targeted transfer, and anticipation of student challenges illustrates FS's deliberate and student-centered approach to analogical reasoning in mathematical problem design.

The differences in analogical reasoning strategies between high-ability (MS) and low-ability (FS) prospective teachers based on the ARM framework are presented in Table 1.

Table 1. Analogical Reasoning of Prospective Teachers based on ARM Framework

ARM Framework Aspects	High Ability Prospective Teachers (MS)	Low Ability Prospective Teachers (FS)
Adapting	Made several key adaptations to simplify the complex problem. Created two different analog problems.	Made drastic simplifications by removing non-essential elements. Added concrete measurements to make calculations easier. Presented analog problems with pictures to enhance understanding. Created a single, concrete analog problem.
Associating	Created analogy problems that help students connect the total angles of a shape with the area of a circle or sector and explain how these relate to unknown individual angle measures.	Associated the concepts of the area of a sector and area comparison as key ideas students need to understand. Focused primarily on calculating the area of the sector and comparing it to other regions.
Distinguishing	Identified specific difficulties that middle school students might face when dealing with complex problems. These included challenges with verbal representation without pictures, relating the area of a sector to the sum of angles in a triangle, lack of specific angle measures, and encountering unfamiliar problem types.	Recognized differences in the number of plane figures (triangles and circles vs. only circles) and in problem representation (verbal vs. pictorial) between complex and analog problems. Considered students' difficulties in translating verbal problems and understanding the concept of comparison.

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Exporting	Retained the core conceptual relationship between the area of a sector of a circle and the sum of the interior angles of a polygon (triangle or square) in the analogous problems. Also transferred the idea of comparing the areas of circular regions inside and outside polygons.	Exported the concept of area comparison from the complex problem to the analog problem. Added the word "comparison" explicitly to guide students' understanding.
Recalling	Reflected on personal understanding of solving complex problems, relating it to the concepts of the area of a sector and the sum of the angles in a triangle. Considered the types of circle sector problems typically encountered by junior high school students.	Considered students' difficulties in translating verbal problems into pictures, determining areas without measurements, and locating circles without information on triangle type. Also took into account students' challenges in understanding the concept of comparison.

Discussion

The prospective teachers' analogical constructions highlighted the practical application of analogical reasoning in pedagogical contexts. Their various strategies for simplifying complex problems and their focus on addressing anticipated student difficulties underscore the potential use of analogical tasks in mathematics education. The diversity of their approaches also suggests that there is no single "correct" way to create analog problems; rather, their effectiveness depends on the specific learning objectives, and the challenges students may face. This aligns with research emphasizing the importance of understanding students' analogical reasoning processes (Hicks, 2022; Hasan et al., 2024; Hicks, 2024).

Overall, high-ability (MS) prospective teachers demonstrated a more comprehensive approach to analogical reasoning. They were able to identify various student difficulties (Kristayulita et al., 2020) and create multiple analog problems that included both visual and conceptual adaptations. In contrast, lower-ability prospective teachers (FS) adopted a simpler approach, focusing on one very concrete and easily calculable analog problem by removing complex elements such as triangles. Both groups succeeded in simplifying the complex problem into a more comprehensible analog problem through ARM activities, emphasizing a student-centered approach to facilitate the transfer of understanding (Hicks, 2024).

High-ability prospective teachers demonstrated more comprehensive and student-centered analogical reasoning strategies. In the *Distinguishing* activity, they identified specific difficulties that junior high school students may encounter, such as challenges in representing verbal problems without pictures, connecting the concept of a sector's area with the sum of the angles in a triangle, dealing with missing angle measures, and addressing problem types that were unfamiliar to students. Their *Recalling* process involved reflecting on their own understanding of the complex problem and on the types of curved-area problems familiar to junior high school students. In *Exporting*, they retained the core conceptual relationship between the area of a sector and the total angles in a polygon, as well as the notion of area comparison. Their *Adapting* strategies included adding visual diagrams, varying the number of circles (one or four), using different polygon shapes (triangle or square), and manipulating angle conditions (known/unknown) to gradually guide students. This approach led to the creation of two distinct analog problems. Their *Associating* activity aimed to help students understand the relationship between the area of a sector and angle proportions, articulating this conceptual analogy explicitly (Hicks, 2022; Hicks & Flanagan, 2024).

In contrast, low-ability prospective teachers (FS) employed more drastic and concrete simplification strategies. During the *Distinguishing* stage, FS also identified student difficulties but focused more on differences in representation (verbal vs. pictorial), the need for concrete numerical data, and challenges in determining circle positions without triangle information. Their *Recalling* involved remembering their own methods for solving complex problems by sketching and applying concepts such as the sum of triangle angles and the area of a sector. In *Exporting*, FS explicitly transferred area comparison questions and added the word “comparison” to enhance clarity. Their *Adapting* strategies involved reducing the number of circles to one, omitting triangles entirely, and adding concrete measures for radius and central angle. They also included pictures to aid comprehension. As a result, FS produced a single analog problem that was very simple and easy to calculate. Their *Associating* activity primarily emphasized the concepts of a circle’s area and area comparison as key ideas students need to understand (Hicks, 2024; Hicks & Flanagan, 2024).

The differences in these strategies, revealed through the analysis of ARM activities, highlight distinct approaches teachers may take in designing learning materials using analogies. The MS approach—creating varied and progressive analogies—has the potential to address a wider range of student difficulties and foster deeper conceptual understanding of structural relationships. Meanwhile, the FS approach, which greatly simplifies problems into highly concrete examples, can serve as an accessible entry point for students who struggle with representation or computation. Notably, students often find it challenging to construct their own analogies (Tzuriel, 2024), emphasizing the critical role of teachers in preparing effective analogies. These findings suggest that a combination of both approaches—identifying diverse student difficulties (as demonstrated by MS) and providing concrete, accessible examples (as modeled by FS)—may represent the most effective strategy for employing analogical reasoning in mathematics learning. This balance underscores the importance of teachers’ roles as problem designers in mathematics education (Hasan et al., 2024; Mutia et al., 2023).

CONCLUSION

This study concludes that prospective mathematics teachers employ analogical reasoning by identifying and addressing specific student difficulties—such as challenges in visual representation, reliance on numerical information, and limited understanding of area comparison—to transform complex problems into more accessible analog problems. Through distinguishing structural elements, recalling relevant concepts, exporting core ideas, adapting problem contexts, and associating key concepts with familiar experiences, they facilitated students' conceptual transfer from simpler analog problems to more complex ones. The findings address the research question by demonstrating that analogical reasoning, when designed with visual supports and familiar question structures, fosters a student-centered learning process and enhances students' problem-solving readiness.

High-ability prospective teachers adopted a comprehensive approach by identifying various student difficulties and developing multiple analog problems with visual and conceptual adaptations. Conversely, lower-ability prospective teachers opted for a simpler approach, focusing on one concrete and easily solvable analog problem by removing complex elements such as triangles. It is recommended that teachers combine these two approaches—first identifying specific student difficulties and then designing a range of analog problems to target different challenges. At the same time, providing concrete and straightforward examples can help students build a solid conceptual foundation before tackling more complex problems. This balanced approach can enhance the effectiveness of mathematics learning through analogical reasoning.

Future research could involve a larger and more diverse sample of prospective teachers to explore analogical reasoning across various mathematical topics and contexts. Longitudinal studies would also be valuable in examining how prospective teachers develop and refine their analog problem construction skills over time, and how these skills influence classroom practices and student learning outcomes.

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